

The Henryk Niewodniczański
INSTITUTE OF NUCLEAR PHYSICS
ul. Radzikowskiego 152, 31-342 Kraków, Poland

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**Non-Uniform Chiral Phase
in Effective Chiral Quark Models**

M. Sadzikowski and W. Broniowski

Abstract

We analyze the phase diagram in effective chiral quark models (the Nambu-Jona-Lasinio model, the σ -model with quarks) and show that at the mean-field level a phase with a periodically-modulated chiral fields separates the usual phases with broken and restored chiral symmetry. A possible signal of such a phase is the production of multipion jets travelling in opposite directions, with individual pions having momenta of the order of several hundred MeV. This signal can be interpreted in terms of disoriented chiral condensates.

Keywords: hot and dense matter, phase transitions, chiral quark models, disoriented chiral condensate

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The problem of phase transitions in hadronic matter is one of the most important issues in strong-interaction physics. It has been studied theoretically for many years with a variety of methods, with the results gaining importance at the approach of the operation of RHIC. We now believe that in the $(T-\mu)$ diagram, where T denotes the temperature and μ the baryon chemical potential, one can recognize a quite complicated structure: phases with broken/restored chiral symmetry, confined/deconfined phases, color superconducting phase (in the case of two light flavors) [1], or color-flavor locked phase (in the case of three light flavors) [2]. In certain limits detailed features of phase transitions are known from first-principle QCD calculations (color superconducting gaps at asymptotically-large baryon densities [3]), or general symmetry considerations (*e.g.* the universality class of the chiral phase transition [4] in the massless limit). Lattice calculations provide information for finite-temperature QCD at zero baryon density [5]. However, these “exact” methods are inapplicable to the case of moderate baryon densities (up to a few times the nuclear saturation density), where out of necessity we have to rely on models [6, 7]. In fact, a lot of our experience comes from model calculations, just to mention the expectation that chiral symmetry is eventually restored as the baryon density is increased.

In this letter we consider the chiral phase structure for a class of low-energy chiral models: the Nambu–Jona-Lasinio model [8] and the linear σ -model with quarks [9]. These models have been used extensively over the past years to describe low-energy properties of mesons and baryons. They have also been used to describe dense and hot system of the Fermi gas of quarks [6], in particular to study chiral restoration at high T or μ . What is usually claimed is that at the mean-field level (synonymous to one-quark-loop or leading- N_c) the model has two phases: with broken chiral symmetry, *i.e.* large dynamically-generated quark mass M , which appears at low T and μ , and with restored chiral symmetry, with massless (or almost massless) quarks, at higher values of T or μ . We argue that the situation is much more complicated if one allows for more general mean-field solutions of the model. We show that for a certain range of T and μ there appears *a non-uniform phase with broken rotational and isospin symmetry*. In the T - μ diagram this phase separates the usual chiral-symmetry-broken phase from the chiral-symmetry-restored phase. The results for the linear σ -model with quarks at $T = 0$ were obtained a long time ago by Kutschera, Kotlorz, and one of us (WB) [10]. Here we generalize the calculation to finite temperatures, as well as apply it to the Nambu–Jona-Lasinio model.

The Lagrangian densities of the σ -model and of the partially-bosonized Nambu–Jona-Lasinio model have the form $\bar{\psi} [i\cancel{\partial} - g(\sigma + i\gamma_5\tau \cdot \pi)] \psi + L_m(\sigma, \pi)$, where ψ denotes the quark fields, g is the quark-meson coupling constant, and the mesonic part $L_m(\sigma, \pi)$ is specific to the model. We consider spatially non-uniform, time-independent chiral fields:

$$\sigma(\vec{x}) = \frac{M}{g} \cos(\vec{q} \cdot \vec{x}), \quad \pi_0(\vec{x}) = \frac{M}{g} \sin(\vec{q} \cdot \vec{x}), \quad \pi_{\pm}(\vec{x}) = 0, \quad (1)$$

where M and \vec{q} are parameters, which are determined dynamically.

Such an ansatz has been considered for the first time in Refs. [11] in context of the *pion condensation* in nuclear matter. In the case of vanishing \vec{q} we recover $\sigma = \frac{M}{g}$, $\vec{\pi} = 0$. The Dirac equation with fields (1) can be solved exactly [11, 10], which facilitates greatly practical [10, 12] and theoretical [13] applications. The quark spectrum has two branches:

$$E_{\pm}(\vec{p}) = \sqrt{M^2 + \vec{p}^2 + \frac{1}{4}\vec{q}^2 \pm \sqrt{M^2\vec{q}^2 + (\vec{q} \cdot \vec{p})^2}}, \quad (2)$$

where \vec{p} labels the momentum of the quark. The basic quantity of our study is the grand thermodynamical potential density $\Omega(T, \mu; M, \vec{q})$. Its global minimum with respect to M and \vec{q} at fixed T and μ determines the ground state of the system subject to conservation of baryon number. We stress that our calculation is self-consistent, *i.e.* both quark and meson fields are solutions to the equations of motion. At the mean-field (one-quark-loop) level we have explicitly

$$\begin{aligned} \Omega(T, \mu; M, \vec{q}) &= \Omega_0(M, \vec{q}) \\ &- 2N_c T \sum_{i=\pm} \int \frac{d^3 p}{(2\pi)^3} \ln \left[\left(1 + e^{(\mu - E_i(\vec{p}))/T} \right) \left(1 + e^{-(\mu + E_i(\vec{p}))/T} \right) \right], \end{aligned} \quad (3)$$

The medium part of Ω describes the ideal gas of quarks with the spectrum (2). The vacuum part, Ω_0 , depends on the model considered. In the σ -model

$$\Omega_0^\sigma(M, \vec{q}) = \frac{m_\sigma^2 F_\pi^2}{8} \left(M^2/M_0^2 - 1 \right)^2 + \frac{F_\pi^2}{2} \left(M^2/M_0^2 \right) \vec{q} \cdot 50^2, \quad (4)$$

where the parameter $M_0 = gF_\pi$ is the vacuum value of the constituent quark mass, $F_\pi = 93\text{MeV}$ is the pion decay constant, and m_σ is the vacuum value of σ -meson mass, treated as a model parameter. The first term in (4) comes from the Mexican Hat potential, while the second term is the kinetic energy of the meson fields. In the Nambu–Jona-Lasinio model we have

$$\Omega_0^{NJL}(M, \vec{q}) = (M^2 - M_0^2)/(4G) + V(M) + \frac{F_\pi^2}{2} \left(M^2/M_0^2 \right) \vec{q} \cdot 50^2 + \mathcal{O}(q^4), \quad (5)$$

where G is the four-quark coupling constant, and $V(M)$ is the Dirac sea contribution to the energy for the case $\vec{q} = 0$. Here we adopt the simple 3-momentum regulator, hence $V(M) = -2N_c/\pi^2 \int_0^\Lambda dp p^2 \left(\sqrt{p^2 + M^2} - \sqrt{p^2 + M_0^2} \right)$, where Λ is the cut-off parameter. In the calculation presented below we use the parameters of Ref. [14]: $G = 5.01\text{GeV}^{-2}$, $\Lambda = 650\text{MeV}$. Equation (5) has been gradient-expanded in powers of \vec{q} . The coefficient of the quadratic term is model independent. Higher-order terms depend on the value of Λ . However, their contribution is small for moderate values of \vec{q} [13] and we can safely drop them. With this simplification the two models become very similar. The same conclusion holds for other variants of the Nambu–Jona-Lasinio model, which differ by the regularization method. In fact, the basic difference between the models is the difference of values of Ω_0 at the minimum and at the point $M = 0$, $\vec{q} = 0$. The larger this difference, the more difficult it is to restore chiral symmetry.

The essence of the dynamics of our system can be understood as follows: as we increase the chemical potential, it becomes favorable for the system to develop a non-zero \vec{q} . This is because the quark energies on the E_- branch lower with \vec{q} , and the energy gain overcomes the repulsion of the meson kinetic term in Ω_0 . Thus, for certain T and μ the ground state has finite \vec{q} . At the further increase of μ the mass of the quark drops to 0, and chiral symmetry is restored. The phase diagram for the Nambu–Jona-Lasinio model with parameters of Ref. [14] is shown in Fig. 1(a). We can see three phases: at low μ and T we have the uniform chirally-broken phase, with $M > 0$ and $\vec{q} = 0$, wrapped around it is the phase with $M > 0$ and $\vec{q} \neq 0$, and at high μ or T we find the chirally-restored phase, with $M = 0$. The lower line in Fig. 1(a) describes the *first-order* phase transition between the uniform and non-uniform chirally broken phases. The discontinuities in the M and \vec{q} parameters can be read-off from Fig. 1(c). Correspondingly, in the baryon density – temperature diagram (Fig. 1(b)) we notice the region of equilibrium of the two phases (the region between the two lower lines). The upper curve in Fig. 1(a) shows the *second-order* phase transition between the non-uniform chirally broken phase to the chirally-restored phase. At this phase transition the order parameter M vanishes at non-zero \vec{q} . Interestingly, the character of the transition between the uniform and non-uniform chiral phases depends on the parameters of the models. In the σ -model for $m_\sigma = 800\text{MeV}$ we find a very similar diagram as in Fig. 1. However for $m_\sigma = 1200\text{MeV}$ the phase transition is second-order. It changes character around $m_\sigma = 1040\text{MeV}$.

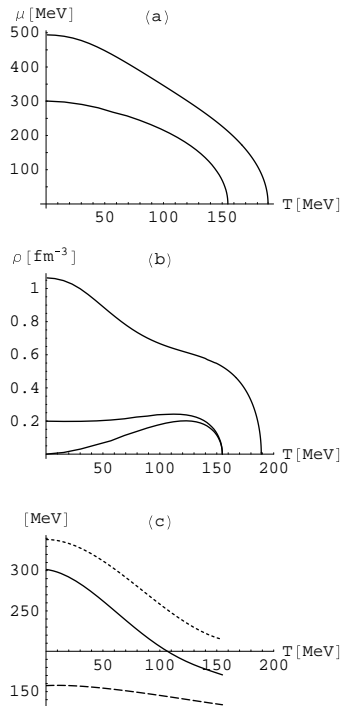


Fig. 1: (a) The T - μ phase diagram for the Nambu–Jona-Lasinio model with parameters of Ref. [14]. The lower line separates the uniform and non-uniform chirally-broken phases (first-order phase transition), and the upper curve separates the non-uniform chirally-broken phase from the chirally-restored phase (second-order phase transition). (b) Same as (a) in the T -baryon-density space. The area between the two lower curves is the region of equilibrium of the uniform and non-uniform chirally-broken phases. (c) Values of $|\vec{q}|$ and M in the non-uniform chirally-broken phase at the phase transition (dotted and dashed lines, respectively), and the value of M in the uniform chirally-broken phase at the phase transition (solid line).

There is always the question of applicability and reliability of a model calculation like ours. Firstly, we need to have sufficiently high baryon densities or temperatures to trust the calculation. Physical systems at low densities consist of nucleons, and we need to dissolve them into quarks for the model to be applicable. By geometrical arguments we may expect this to happen at densities of the order of a few nuclear-matter saturation densities. Note that in Fig. 1 this is the region of the non-uniform chirally-broken phase, thus we may hope that the model is valid there. Secondly, the size of the system should be sufficiently large such that our infinite-size calculation is meaningful. It should be larger than $2\pi/|\vec{q}|$ in order to accommodate at least one period of the chiral field. Using the values of $|\vec{q}| \sim 200 - 600$ MeV this gives the minimum size in the range 2 – 6 fm, easily accessible in heavy-ion collisions. In addition, a more elaborate calculation should incorporate meson fluctuations. At low temperatures pions are easily excited due to their small mass, and such effects should certainly be included. Note, however, that inclusion of meson loops in the study of the quark condensate in Ref. [15] modified, but did not invalidate the mean-field results. Finally, the effect of the explicit breaking of chiral symmetry by the finite current quark mass, m , should be incorporated. In studies with $\vec{q} = 0$ [6] this effect was not quantitatively small. Moreover, with finite m chiral restoration is no longer a second-order phase transition, but becomes a smooth cross-over. Another question is whether within the mean-field approximation the solution with fields (1) is the ground state. What we have verified is that for a certain range of T and μ we have a lower-energy state than the uniform

phase. A priori, there may exist yet lower-energy, up-to-now unknown, mean-field solutions.

We conclude with a discussion of a possible signal of the non-uniform chirally broken phase. Suppose that sufficiently large domains of the non-uniform chirally-broken phase are created in the cooling process of the plasma formed in a heavy-ion collision. Such a domain can be treated as a classical source of pions, in full analogy to disoriented chiral condensates (see e.g. [16]). The coherent decay of such a source would lead to a characteristic signal in the form of two multi-pion “jets”, with pion momenta peaked around $\pm\vec{q}$ (in the rest frame of the source), where \vec{q} is taken at the phase-transition point, and is typically $200 - 300 MeV$. Indeed, according to Bjorken’s description, pion quanta $\delta\phi^\alpha(x)$ satisfy the equation $(\square + m_\pi^2)\delta\phi^\alpha(x) = j^\alpha(\vec{x})$, where $j^\alpha(\vec{x})$ is the source, in our case proportional to the classical pion field, i.e. to $\sin(\vec{q}\cdot\vec{x})$. Thus the Fourier transform of $\delta\phi^\alpha$ picks up the components at $\pm\vec{q}$. Finite size of the domain spreads out the distribution, but if the domain has the size of, say, 3 wavelengths $2\pi/|\vec{q}|$, the half-width of peaks is only $\sim 20\%$ of $|\vec{q}|$. Of course, larger domains lead to sharper peaks. If multiple domains contribute, then the signal should be folded with the momentum distribution of the domain and averaged over directions. In Eq. (1) we have used the neutral classical pion field. This case leads to production of neutral pion quanta. However, we can rotate ansatz (1) in isospin, which leads to a degenerate solution. Such a configuration will decay into jets of charged pions. In addition, the quanta of the σ field decay subsequently into pairs of neutral as well as charged pions. A more detailed calculation of the spectrum of pions requires additional assumptions for the dynamics of the phase transitions and is left for a further study. Also, the analysis of the formation and stability of domains is outside of the scope of this paper.

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