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www.ifj.edu.pl/reports/2000.html

Kraków, April 2000

Report No 1844/PH

Two Hagedorn temperatures: a larger one for mesons, a lower one for baryons

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Abstract:

Our experimental work involved reading throught Particle Data Tables[1]. We show that the Hagedorn temperature is larger for mesons than for baryons.

PACS numbers: 14.20.-c, 14.40.-n, 12.40Yx, 12.40Nn

This research is being done in collaboration with Wojciech Florkowski and Piotr Żenczykowski.

The famous Hagedorn hypothesis [2, 3] states that at asymptotically large masses, m, the density of hadronic resonance states, $\rho(m)$, behaves as

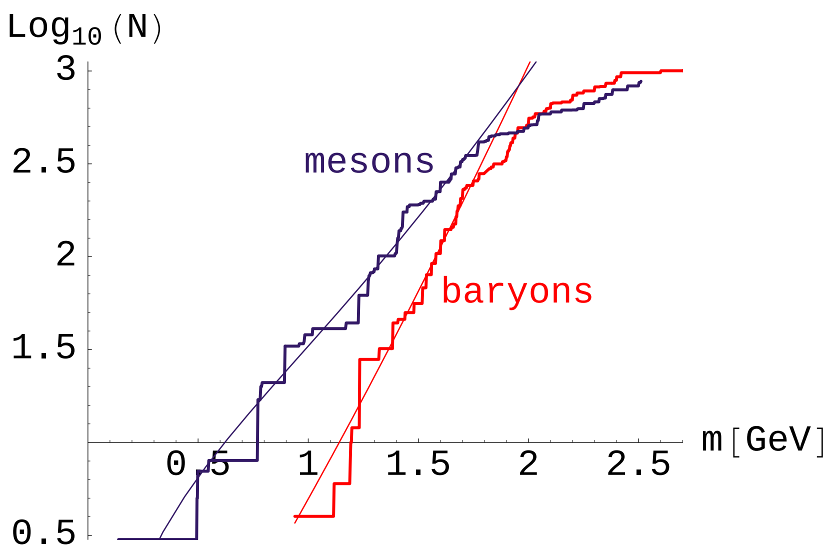

Fig. 1: Cumulants of meson and baryon spectra, and the Hagedorn-like fit.

$$\rho(m) \sim \exp\left(\frac{m}{T_H}\right) \tag{1}$$

The Hagedorn temperature, T_H, is a scale controlling the exponential growth of the spectrum.[1] Ever since hypothesis (1) was made, it has been believed that there is one universal T_H for all hadrons. *Presently available experimental data show that this is not the case* [4].

In Fig. 1 we compare the *cumulants* of the spectrum, defined as the number of states with mass lower than m. The experimental curve is $N_{\rm exp}(m) = \sum_i g_i \Theta(m - m_i)$, where $g_i = (2J_i + 1)(2I_i + 1)$ is the spin-isospin degeneracy of the ith state, and m_i is its mass. The theoretical curve corresponds to $N_{\rm theor}(m) = \int_0^m \rho_{\rm theor}(m')dm'$, where $\rho_{\rm theor}(m) = f(m)\exp(m/T)$, with $f(m)$ denoting a slowly-varying function. A typical choice [3], used in the plot, is $f(m) = A/(m^2 + (500{\rm MeV})^2)^{5/4}$. Parameters T_H and A are obtained with the least-square fit to $\log N_{\rm theor}$. Other choices of $f(m)$ give fits of similar quality. A striking feature of Fig. 1 is the linearity of $\log N$ starting at very low m, and extending till $m \sim 1.8$GeV. Clearly, this shows that (1) is valid in the range of available data.[2] However, the slopes in Fig. 1 are different for mesons and baryons. For the assumed $f(m)$ we get $T_{\rm meson} = 195$MeV and $T_{\rm baryon} = 141$MeV. This means that $T_{\rm meson} > T_{\rm baryon}$, and the inequality is substantial. Although it has been known to researchers in the field of hadron spectroscopy that the baryons multiply more rapidly than mesons, to our knowledge this fact has not been presented as vividly as in Fig. 1. To emphasize the strength of the effect we note that in order to make the meson line parallel to the baryon line, we would have to aggregate ~ 500 additional meson states up to $m = 1.8$MeV as compared to the present number of ~ 400. If Ref. [4] we show that the fitted values of T_H, distinct for mesons and baryons, do not depend on flavor.

Why do mesons and baryons behave so differently? First, let us stress that it is not easy to get an exponentially rising spectrum at all. Take the simplistic harmonic-oscillator model, whose density of states grows as m^{d-1}, with d denoting the number of dimensions. For mesons

[1] T_H need not immediately be associated with thermodynamics, here we are just concerned with the spectrum of particles *per se*.

[2] Above 1.8GeV the data is sparse and we have to wait for this region to be filled in by future experiments.

there is one relative coordinate, hence $\rho \sim m^2$, whereas the two relative coordinates in the baryon give $\rho \sim m^5$. Weaker-growing potentials lead to a faster growth, but fall short of the behavior (1). We know of two approaches yielding behavior (1), both involving combinatorics of infinitely-many degrees of freedom. *Statistical bootstrap* models [2, 5] form particles form clusters of particles, and employ the principle of self-similarity. It can be shown, following *e.g.* the steps of Ref. [6], that the model leads to equal Hagedorn temperatures for mesons and for baryons.[3] Thus the bootstrap idea *is not capable* of explaining the different behavior of mesons and baryons in Fig. 1.

On the other hand, the *Dual String models* [7] do give the demanded effect of $T_{\rm meson} > T_{\rm baryon}$, at least at asymptotic masses. Let us analyze mesons first. The particle spectrum is generated by the harmonic-oscillator operator describing vibrations of the string, $N = \sum_{k=1}^{\infty}\sum_{\mu=1}^{D} k a^\dagger_{k,\mu} a_{k,\mu}$, where k labels the modes and μ labels additional degeneracy, related to the number of dimensions [7]. Eigenvalues of N are composed in order to get the square of mass of the meson, according to the formula $\alpha' m^2 - \alpha_0 = n$, where $\alpha' \sim 1{\rm GeV}^{-2}$ is the Regge slope, and $\alpha_0 \approx 0$ is the intercept. Example: take $n = 5$. This can be made by taking the $k = 5$ eigenvalue of N (this is the leading Regge trajectory, with maximum angular momentum), but we can also take obtain the same m^2 by exciting one $k = 4$ and one $k = 1$ mode, alternatively $k = 3$ and $k = 2$ modes, *etc.* The number of possibilities corresponds to partitioning the number 5 into natural components: 5, 4+1, 3+2, 3+1+1, 2+2+1, 2+1+1+1, 1+1+1+1+1. Partitions with more than one component describe the sub-leading Regge trajectories. With D degrees of freedom each component can come in D different species. Let us denote the number of partitions in our problem as $P_D(n)$. For large n the asymptotic formula for *partitio numerorum* leads to the exponential spectrum according to the formula [8, 7].

$$\rho(m) = 2\alpha' m P_D(n), \quad P_D(n) \simeq \sqrt{\frac{1}{2n}} \left(\frac{D}{24n}\right)^{\frac{D+1}{4}} \exp\left(2\pi\sqrt{\frac{Dn}{6}}\right), \tag{2}$$

where $n = \alpha' m^2$. We can now read-off the mesonic Hagedorn temperature: $T_{\rm meson} = \frac{1}{2\pi}\sqrt{\frac{6}{D\alpha'}}$.

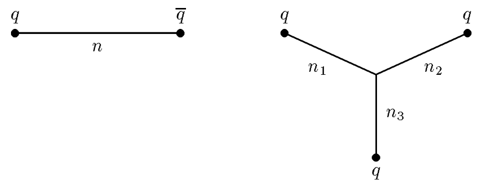

Fig. 2: Meson and baryon string configurations.

Now the baryons: the configuration for the baryon is shown in Fig. 2. The three strings vibrate independently, and the corresponding vibration operators, N, add up. Consequently, their eigenvalues n_1, n_2, and n_3 add up. Thus we simply have a partition problem with 3 times more degrees of freedom than in the meson. The replacement $D \to 3D$ in (2) leads immediately to $T_{\rm baryon} = \frac{1}{2\pi}\sqrt{\frac{2}{D\alpha'}}$, $T_{\rm meson}/T_{\rm baryon} = \sqrt{3}$. We stress the picture is fully consistent with the Regge phenomenology. The leading Regge trajectory for baryons is generated by the excitation of a single string, *i.e.* two out of three numbers n_i vanish. This is the quark-diquark configuration. The subleading trajectories for baryons come in a much larger degeneracy than for mesons, due to more combinatorial possibilities. The slopes of the meson and baryon trajectories are universal, and given by α'. We stress that the "number-of-strings" mechanism described above

[3] Since baryons are formed by attaching mesons to the "input" baryon, the baryon spectrum grows at exactly the same rate as the meson spectrum.

is asymptotic. When applied to the data in the observed region one can, however, obtain very good agreement for mesons with a wide range of the dimensionality parameter D [9]. Baryon slopes can also be descibed properly, however with simplest string models there are too many baryon states. This hints for improvements, *e.g.* the inclusion of the spin-flavor symmetry factors for baryon states. More work needs to be done here.

We summarize the basics of the string mechanism: as m^2 increases, more and more degrees of freedom "wake up". Via partitio numerorum they lead to the exponential growth of the spectrum. The three strings in the baryon bring more degrees of freedom and result in "faster" combinatorics. We have heard many talks in this conference on hadron exotics. If an exotic is a multi-string configuration (generalizations of Fig. 2), then the corresponding spectrum will grow exponentially with the Hagedorn temperature inversely proportional to the square root of the number of strings. For instance, $T_{q\overline{q}q\overline{q}} = \frac{1}{2}T_{\rm meson}$. This is reminiscent of the effect described in Ref. [10].

The author thanks Keith R. Dienes for many profitable e-mail discussions on the issues of hadron spectra and string models, and to Andrzej Białas and Kacper Zalewski for numerous useful comments.

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