

Discovering the real laws of nature with the help of virtual hadron-hadron colliders

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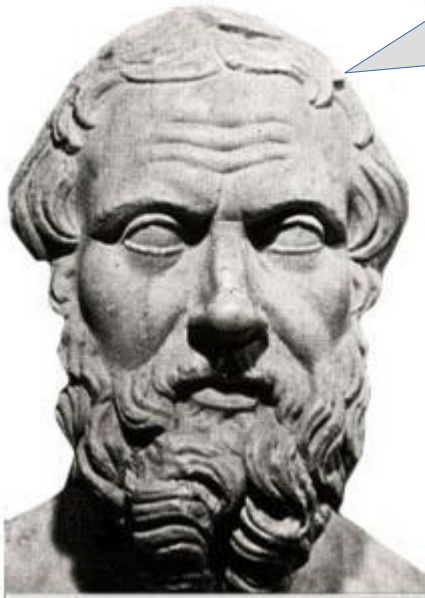


Seminar of the Institute of Nuclear Physics PAN, Kraków, 18th October 2018

1. Motivation
2. What Virtual Colliders are and why they are useful
3. Basic building blocks of Monte Carlo Event Generators
4. Summary and outlook

Motivation: Fundamental questions and the first virtual collider

What is the universe made of?



Cheese could be cut in half, then half again, and so on. But eventually there would be a minute piece of cheese that could not be cut in half, not because there was no knife sharp enough, but because the final particle was not something that could be sliced. He called this an **atom**, which means '**uncuttable**' in Greek

LEDERMAN: And you came up with this idea in fifth-century-B.C. Greece?

DEMOCRITUS: Yes, why? Your ideas today are so much different?

LEDERMAN: Well, actually, they're pretty much the same. It's just that we hate the fact that you published first.

"The God Particle: If the Universe is the Answer, what is the Question?"

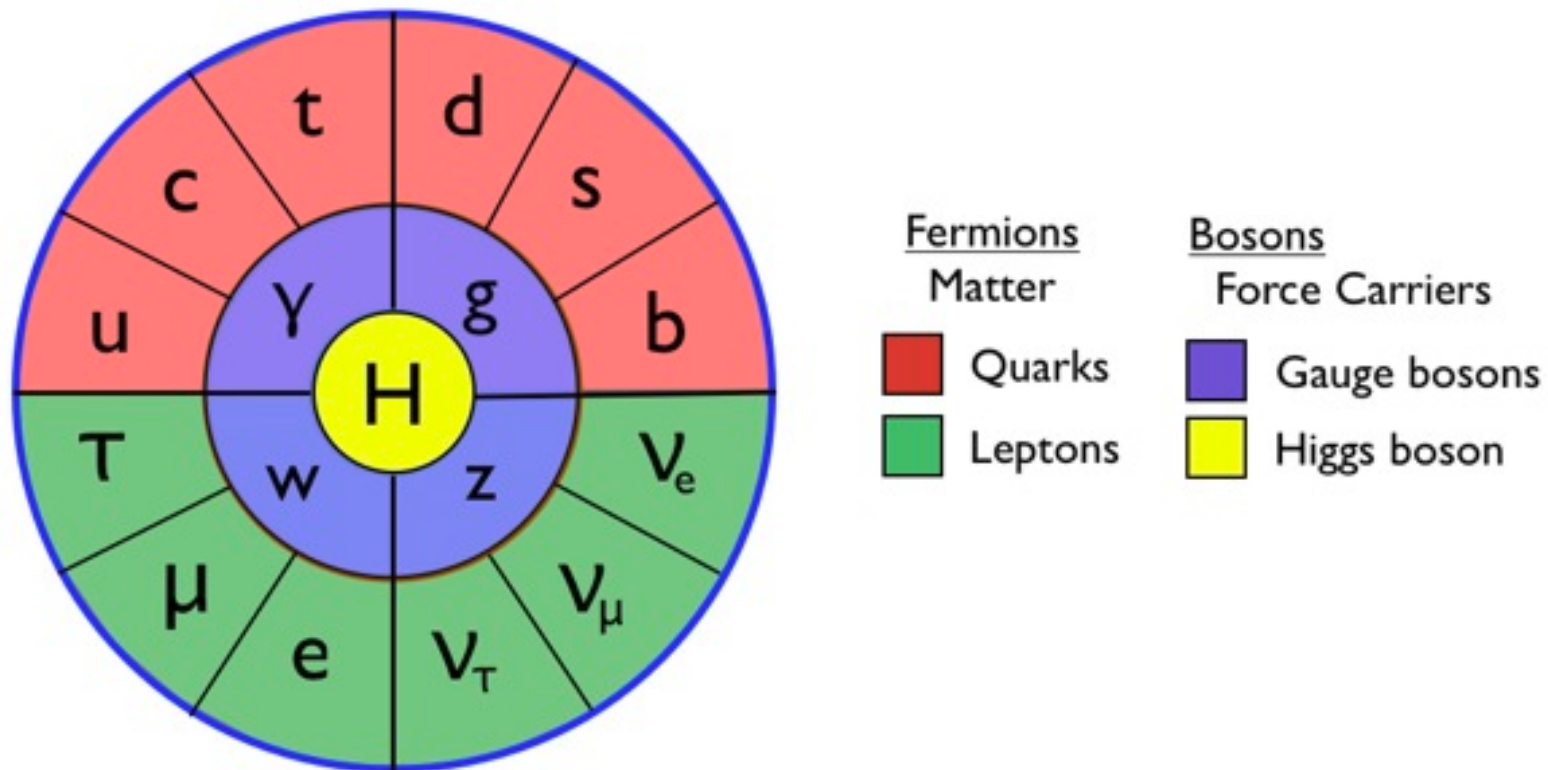
L. Lederman

Motivation: What is the universe made of?



Empedocles suggested that everything was made from 4 basic elements
Until the early 1800's, the 4 basic elements were still widely accepted.

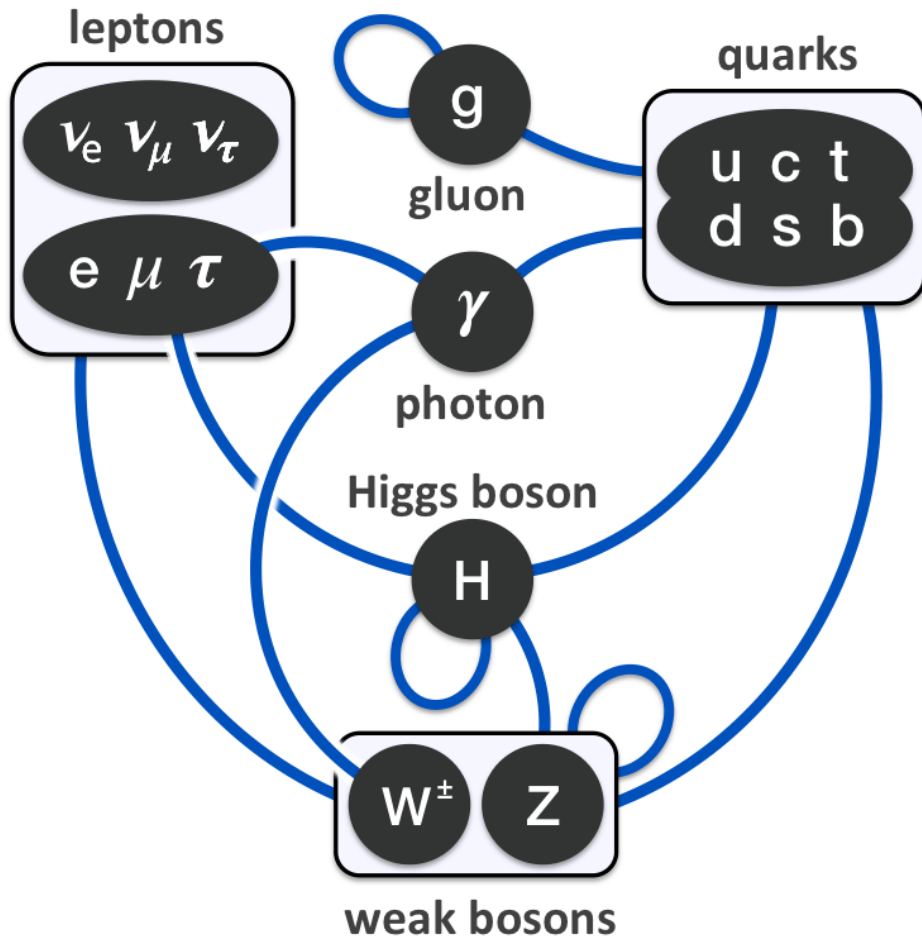
Motivation: What is the universe made of?



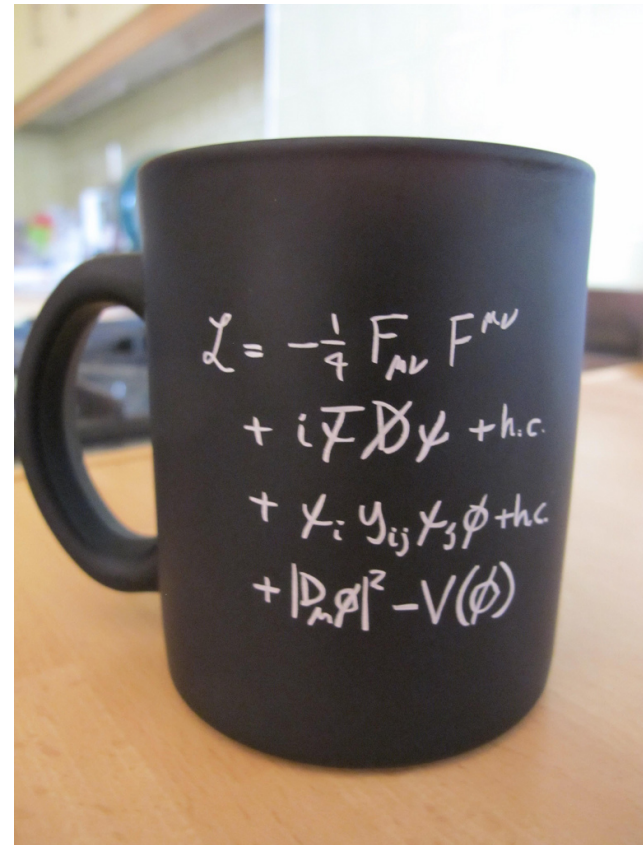
Particles of the Standard Model

Motivation: What is the universe made of?

Standard Model Interactions (Forces Mediated by Gauge Bosons)



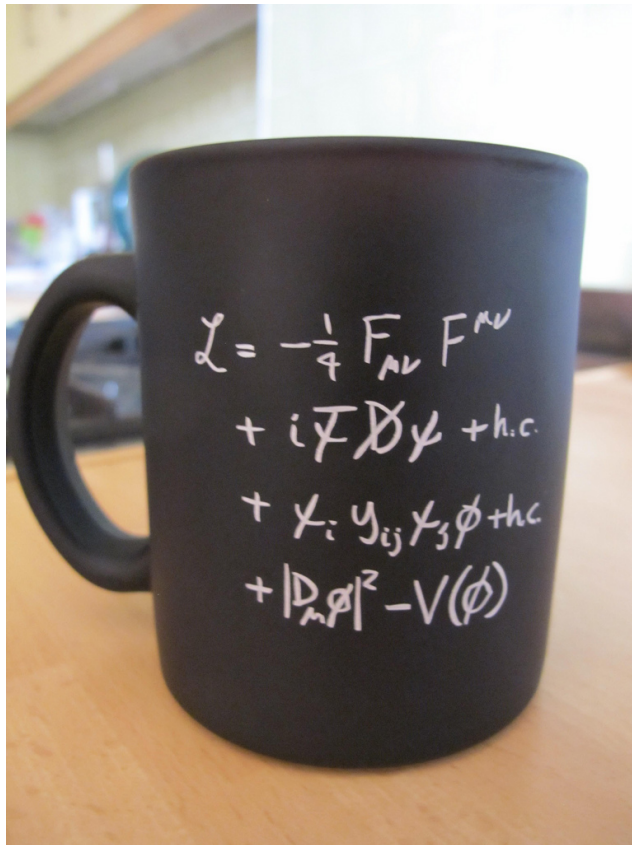
Standard Model Lagrangian



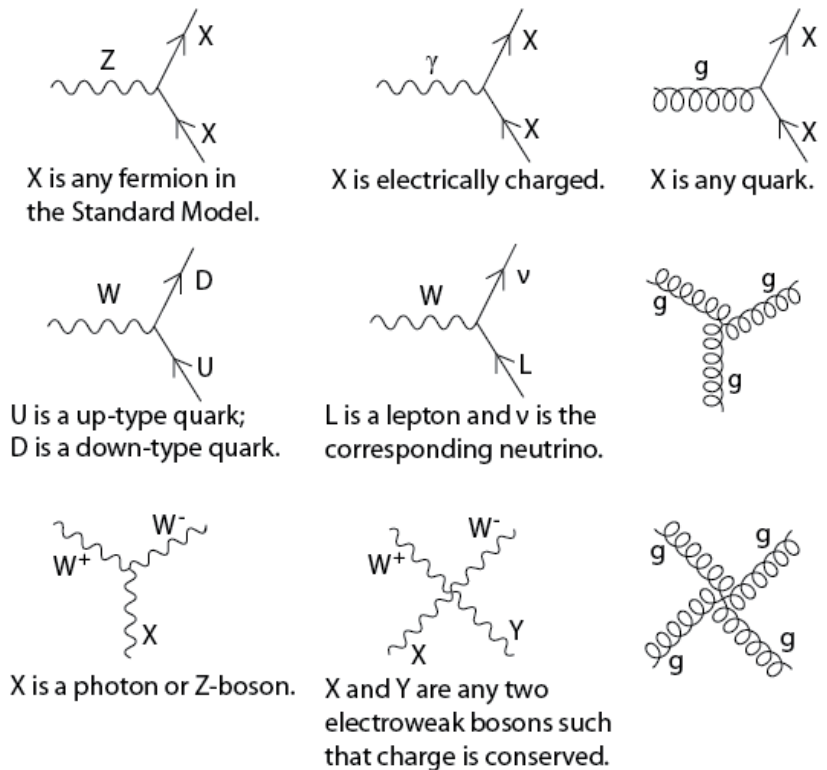
Motivation: What is the universe made of?

Standard Model Interactions (Forces Mediated by Gauge Bosons)

Standard Model Lagrangian

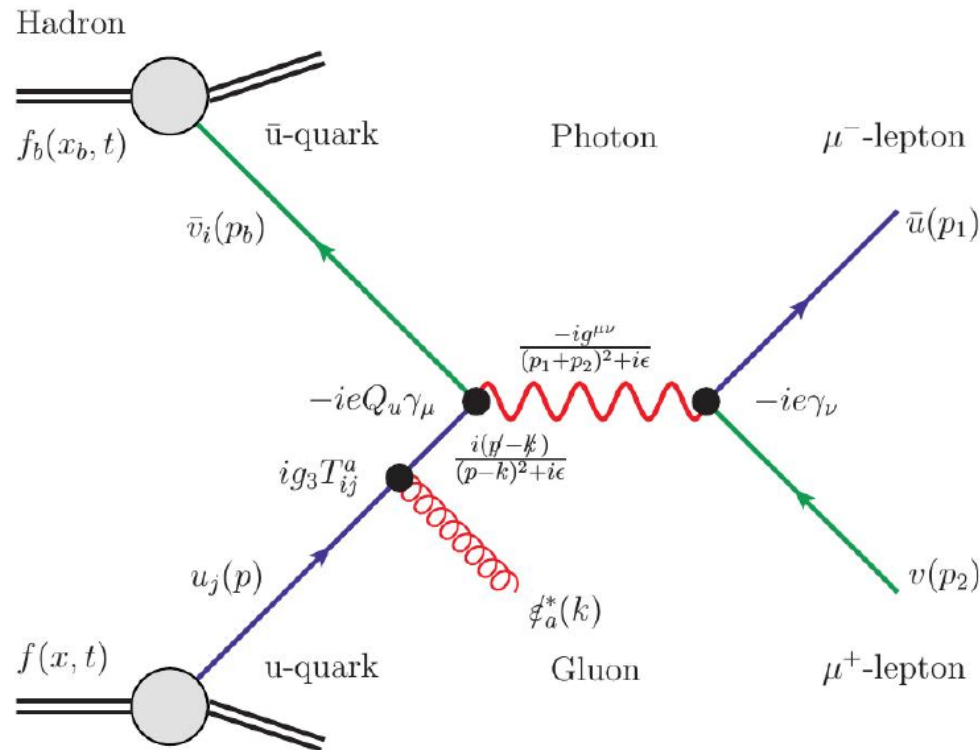


Feynman Diagrams



Motivation: What is the universe made of?

Standard Model Interactions (Forces Mediated by Gauge Bosons)

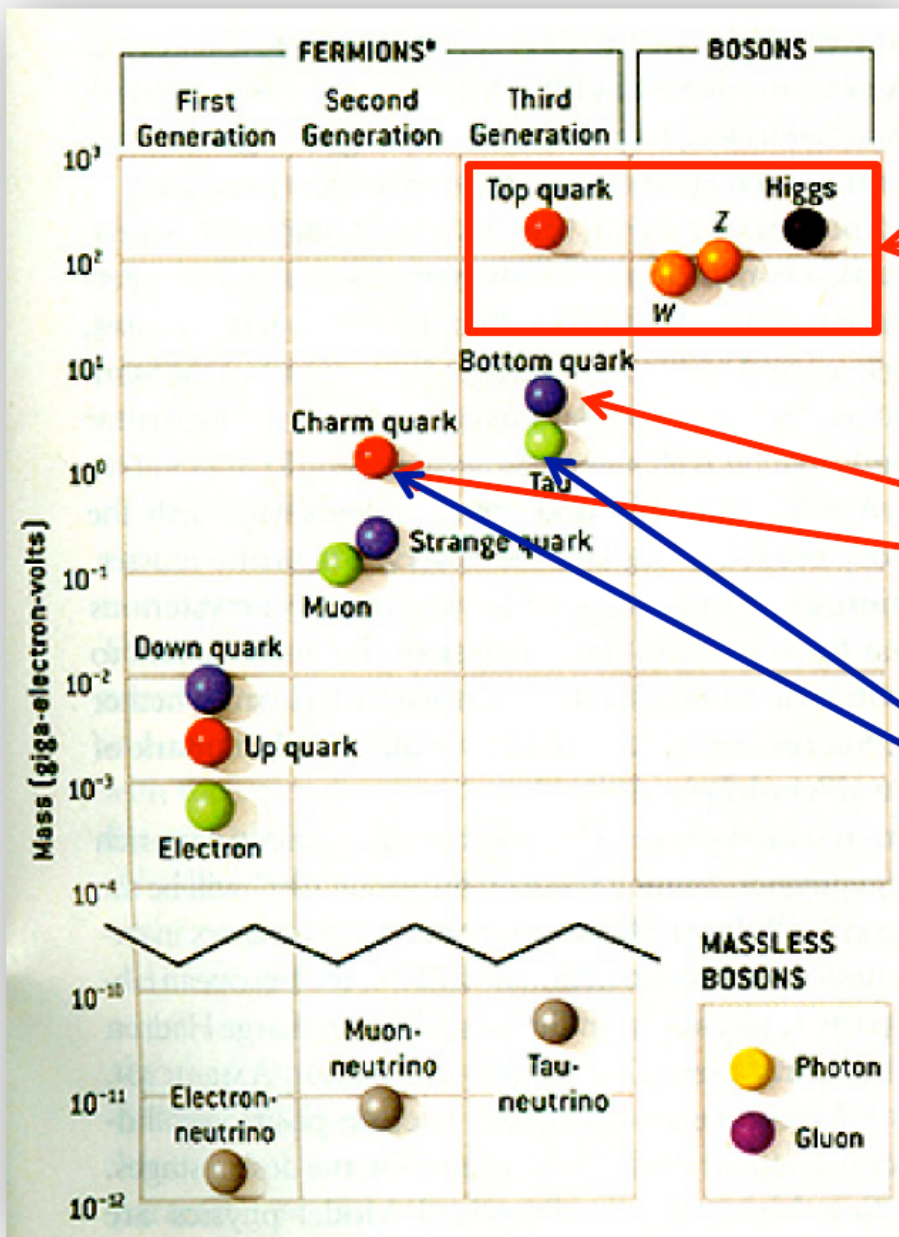


The hadronic cross section is

$$d\sigma(pp \rightarrow \mu^+ \mu^- g + X) = dx dx_b f(x, t) f_b(x_b, t) d\hat{\sigma} \quad , \quad d\hat{\sigma} = \frac{|\mathcal{M}(u\bar{u} \rightarrow \mu^+ \mu^- g)|^2 d\Phi_{n+1}}{4\sqrt{(pp_b)^2}}$$

Motivation: What is the universe made of?

Standard Model Interactions → colliders the key tool for particle physics



Hadron Colliders

W/Z: UA1/UA2 @ SPS

Top: CDF/D0 @ Tevatron

H: ATLAS/CMS @ LHC

Hadron Collisions

b quark: E288 @ FNAL

c quark: pBe @ AGS

ee Collider

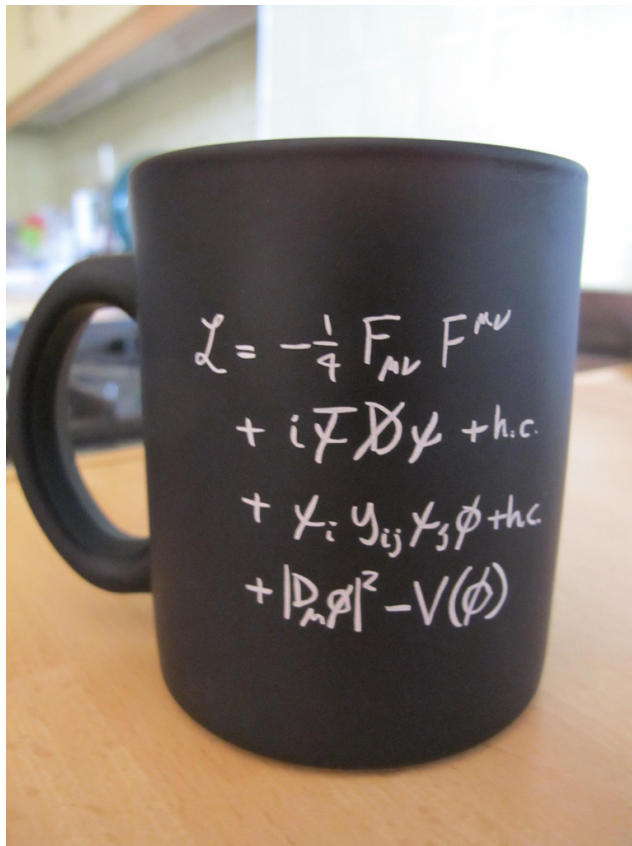
c and τ SPEAR

Motivation: What is the universe made of?

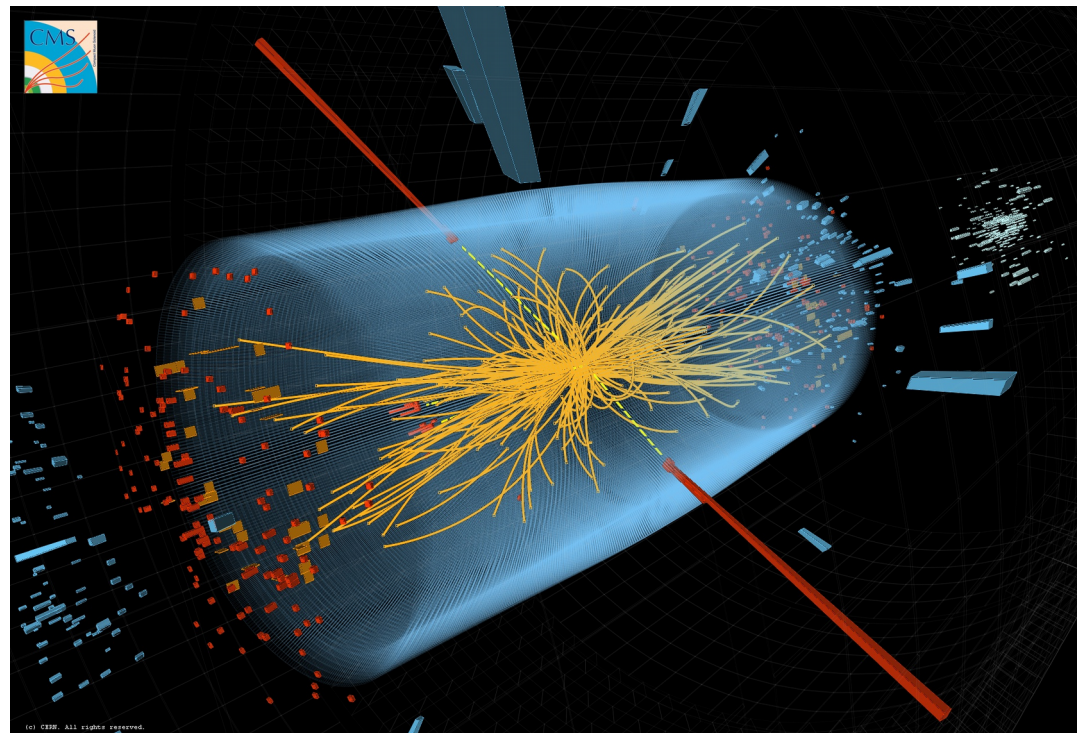
Standard Model Interactions

There is a huge gap between a one-line formula of a fundamental theory, like the Lagrangian of the SM, and the experimental reality that it implies.

Standard Model Lagrangian



Experimental reality



What Virtual Colliders are and why they are useful?

Theory

Lagrangian
Gauge invariance
QCD
Partons
NLO
Resummation
...

DATA MAKES
YOU SMARTER

It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong.

Richard P. Feynman

Fred Oloros

6 September 2013 DESY

Detector simulation
Pions, Kaons, ...
Reconstruction
B-tagging efficiency
Boosted decision tree
Neural network
...

Experiment

What Virtual Colliders are and why they are useful?

Theory

Lagrangian
Gauge invariance
QCD
Partons
NLO
Resummation
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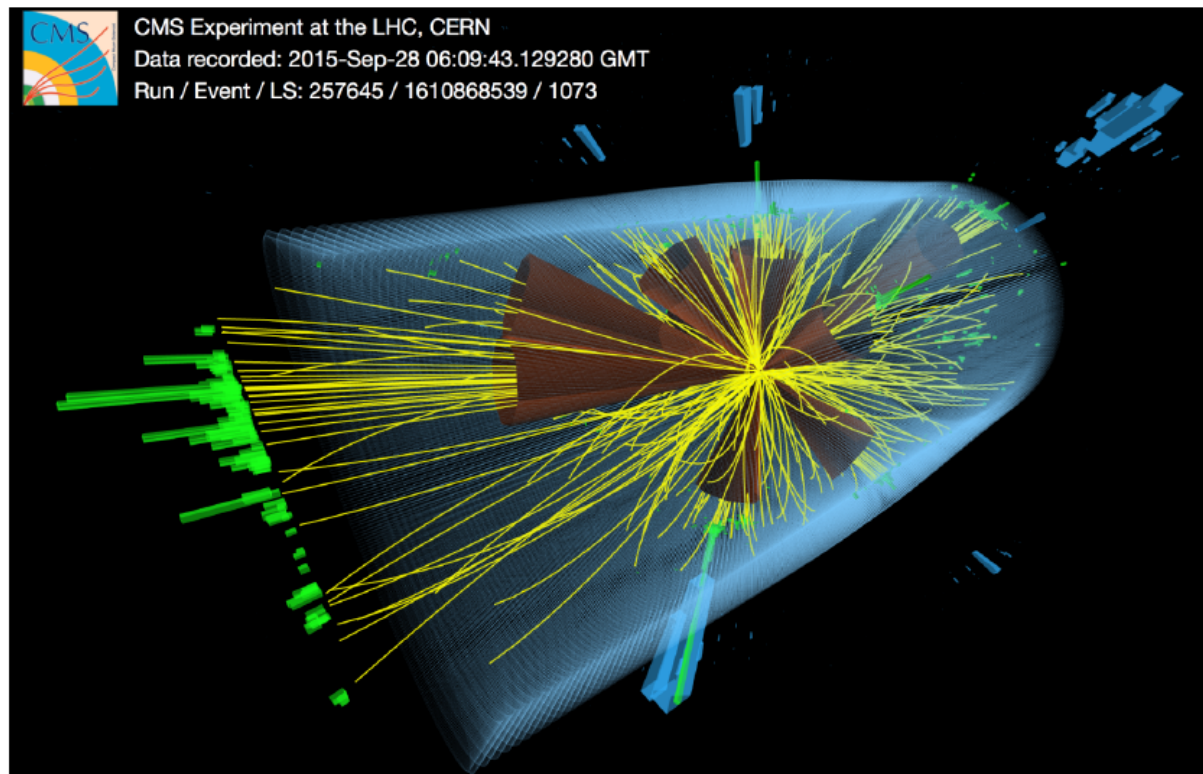


Detector simulation
Pions, Kaons, ...
Reconstruction
B-tagging efficiency
Boosted decision tree
Neural network
...

Experiment

What Virtual Colliders are and why they are useful?

- ▶ General Purpose Monte Carlo (GPMC) event generators are designed to bridge that gap.



- ▶ One can think of a GPMC as a “Virtual Collider” \Rightarrow Direct comparison with the data.
- ▶ Almost all HEP measurements and discoveries in the modern era have relied on GPMC generators, most notably the discovery of the Higgs boson.

Hadron colliders and the importance of strong interactions

Relative strength of the forces at 10^{-15}m (= proton radius):

Strong : Electromagnetic : Weak
1 : 1 / 100 : 1 / 10000

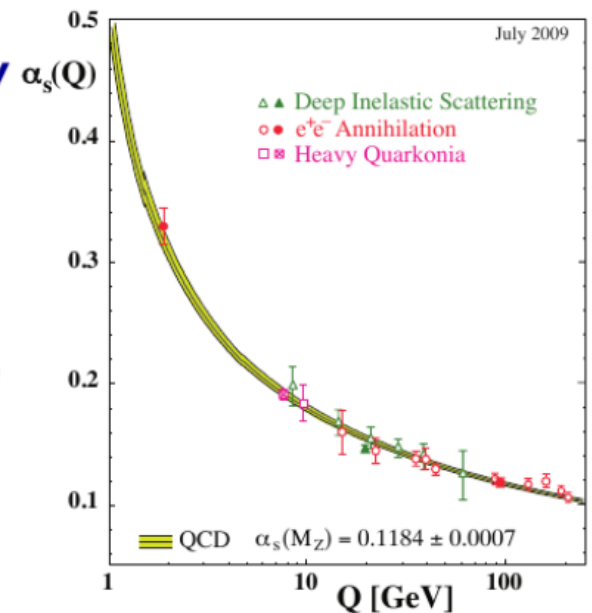
QCD: Quantum field theory of strong interactions

(C.N. Yang, R. Mills; H. Fritzsch, M. Gell-Mann, H. Leutwyler)

- ▶ interaction carried by gluons acting on quarks and gluons
- ▶ QCD-charge: colour : of three types (`colours`: red, blue, green)

QCD coupling strength α_s depends on energy

- ▶ low energy (= long distance or time)
 - α_s is large (confinement): non-perturbative regime of QCD
- ▶ high energy (= short distance or time)
 - α_s is small (asymptotic freedom): perturbative regime of QCD



Particle Data Group

Complex structure of Quantum Chromodynamics – three faces of QCD

Perturbative: $\alpha_s \ll 1$

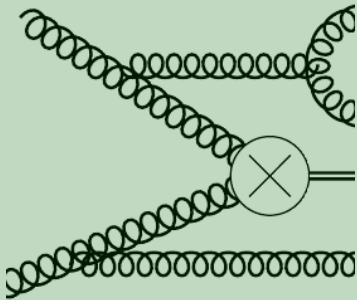
$$\sigma = \sigma_0 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \alpha_s^3 \sigma_3 \dots$$

$$\sigma_0 > \alpha_s \sigma_1 > \alpha_s^2 \sigma_2 > \alpha_s^3 \sigma_3 \dots$$

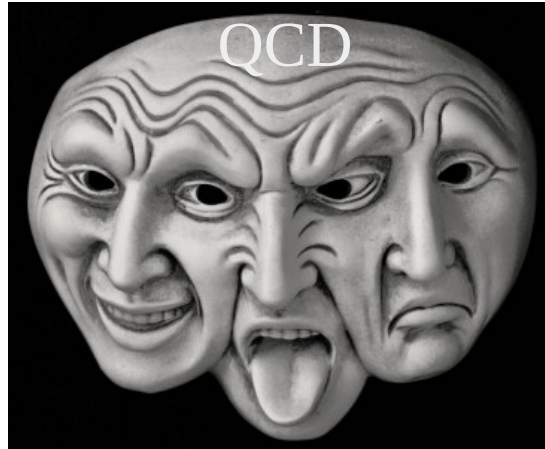
LO NLO NNLO N3LO

State of the art:

“Higgs boson gluon-fusion production in N3LO QCD” *Phys. Rev. Lett.* 114, 212001 (2015)



Example of one of hundreds of diagram

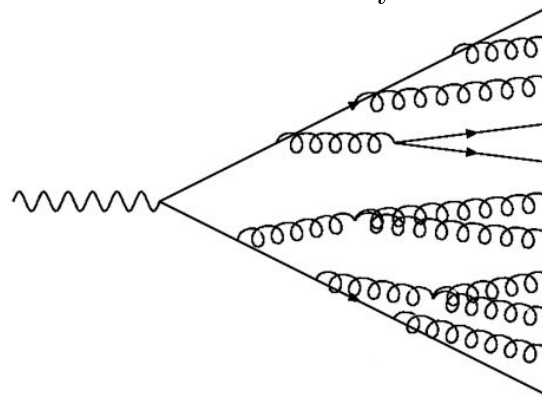


Perturbative resummation:

- enhanced terms

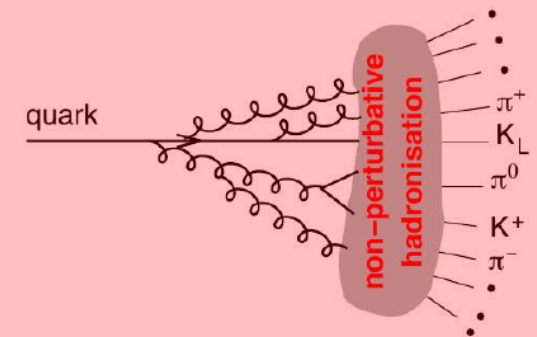
$$\sigma_i \supset L^i$$
$$\sigma_0 \sim \alpha_s L \sim \alpha_s^2 L^2 \sim \alpha_s^3 L^3 \dots$$

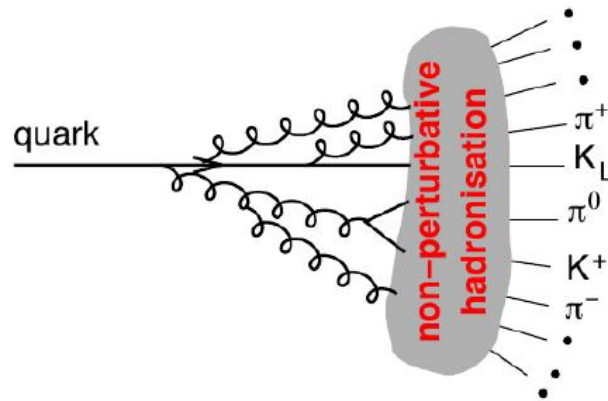
- Resum them $\sum_i \alpha_s^i L^i$



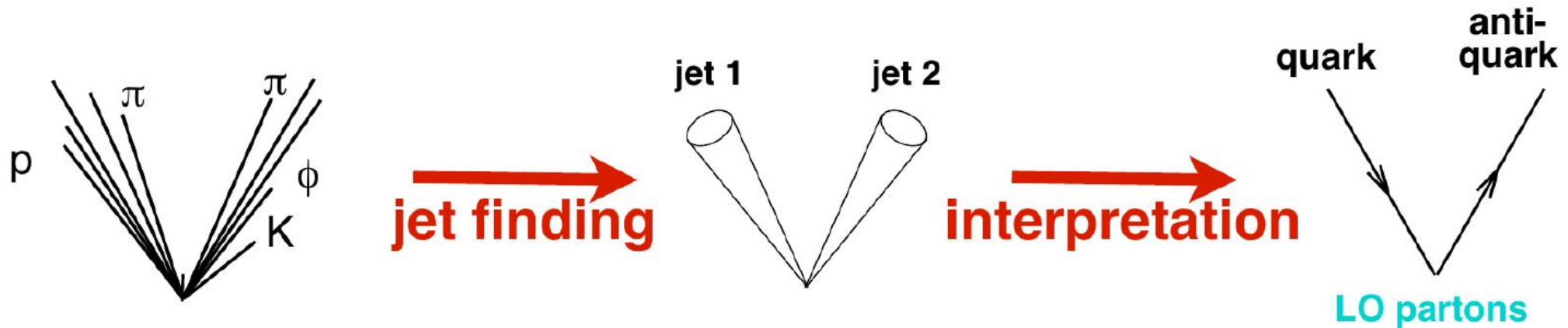
Non-Perturbative: $\alpha_s \gg 1$

- Perturbative techniques break down
- Non-perturbative models inspired by physical motivations
- Lattice QCD?

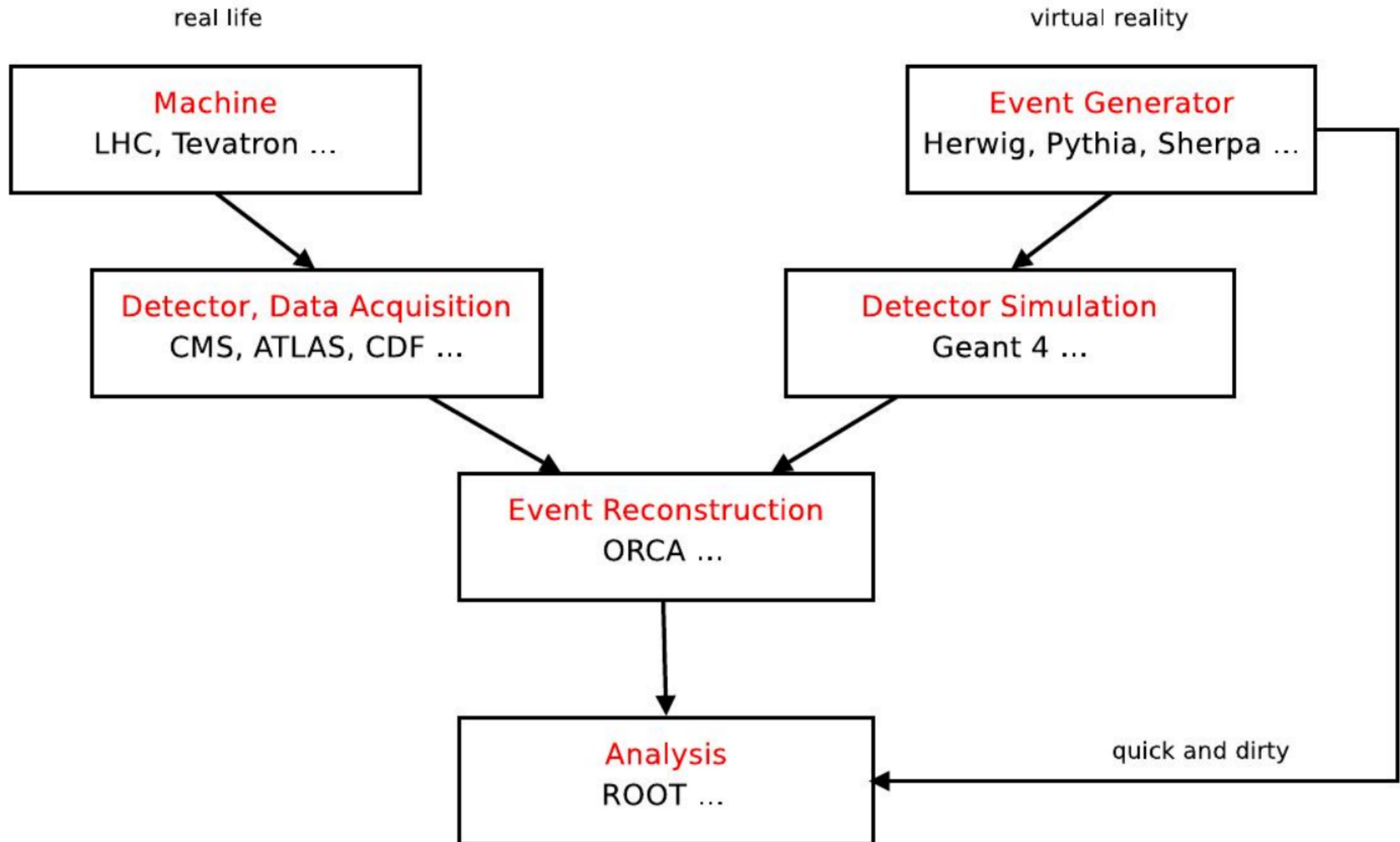




How we “reconstruct” jets



What Virtual Colliders are and why they are useful?



What do MC event generators do?

- ▶ An “event” is a list of particles (pions, protons, ...) with their momenta.
- ▶ The MCs generate events.
- ▶ The probability to generate an event is proportional to the (approximate!) cross section for such an event.
- ▶ Calculate Everything \sim solve QCD (1M \$ prize) \rightarrow requires compromise!
- ▶ Improve lowest-order perturbation theory, by including the “most significant” corrections \rightarrow complete events (can evaluate any observable you want)

The Workhorses: What are the Differences?

All offer convenient frameworks for LHC physics studies, but with slightly different emphasis:

PYTHIA: Successor to JETSET (begun in 1978). Originated in hadronization studies: Lund String.

HERWIG: Successor to EARWIG (begun in 1984). Originated in coherence studies: angular ordering parton shower. Cluster model.

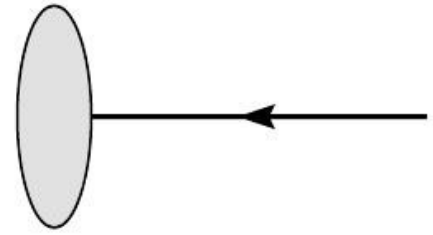
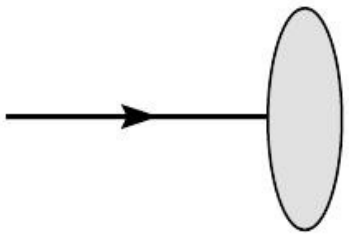
SHERPA: Begun in 2000. Originated in “matching” of matrix elements to showers: CKKW.



Disclaimer: I am an author of Herwig so I will focus on this virtual collider

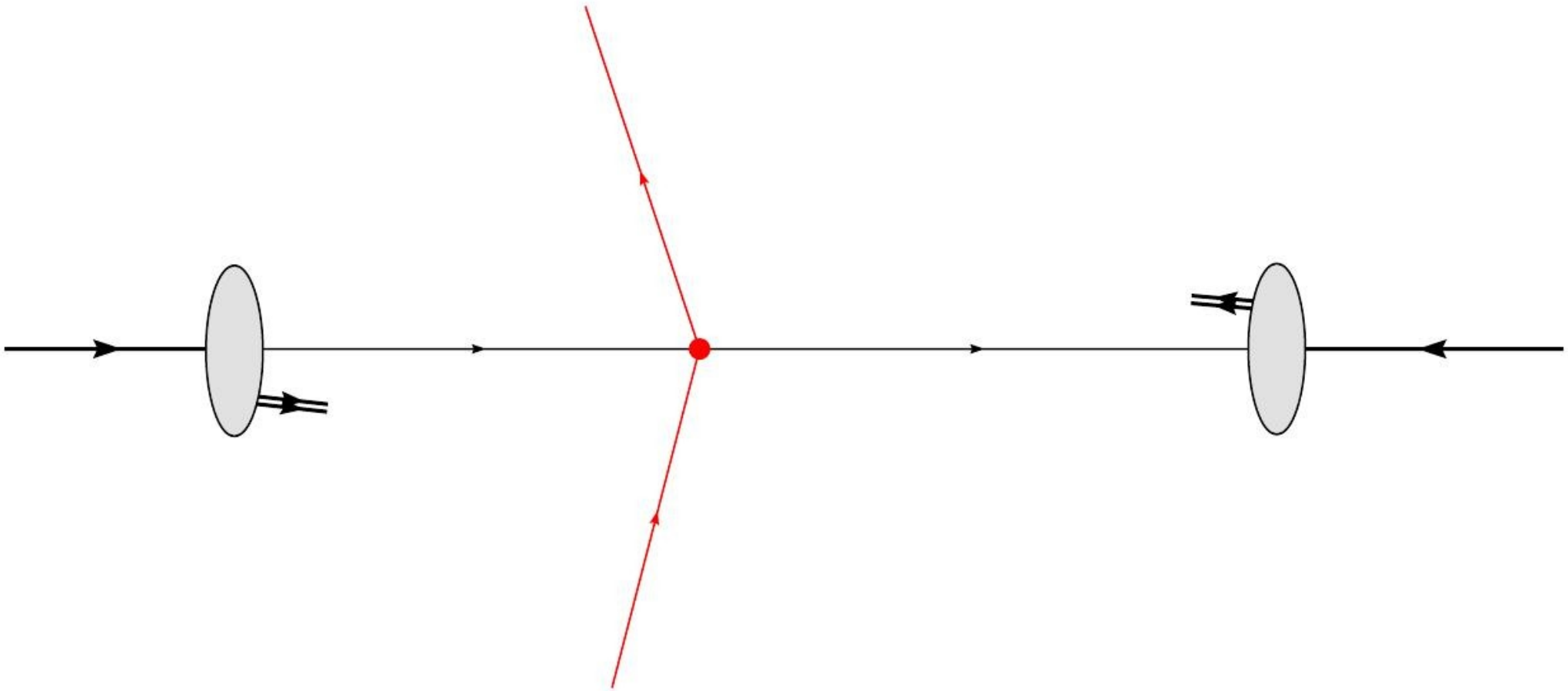
Basic building blocks of Monte Carlo Event Generators

Parton Distribution Function



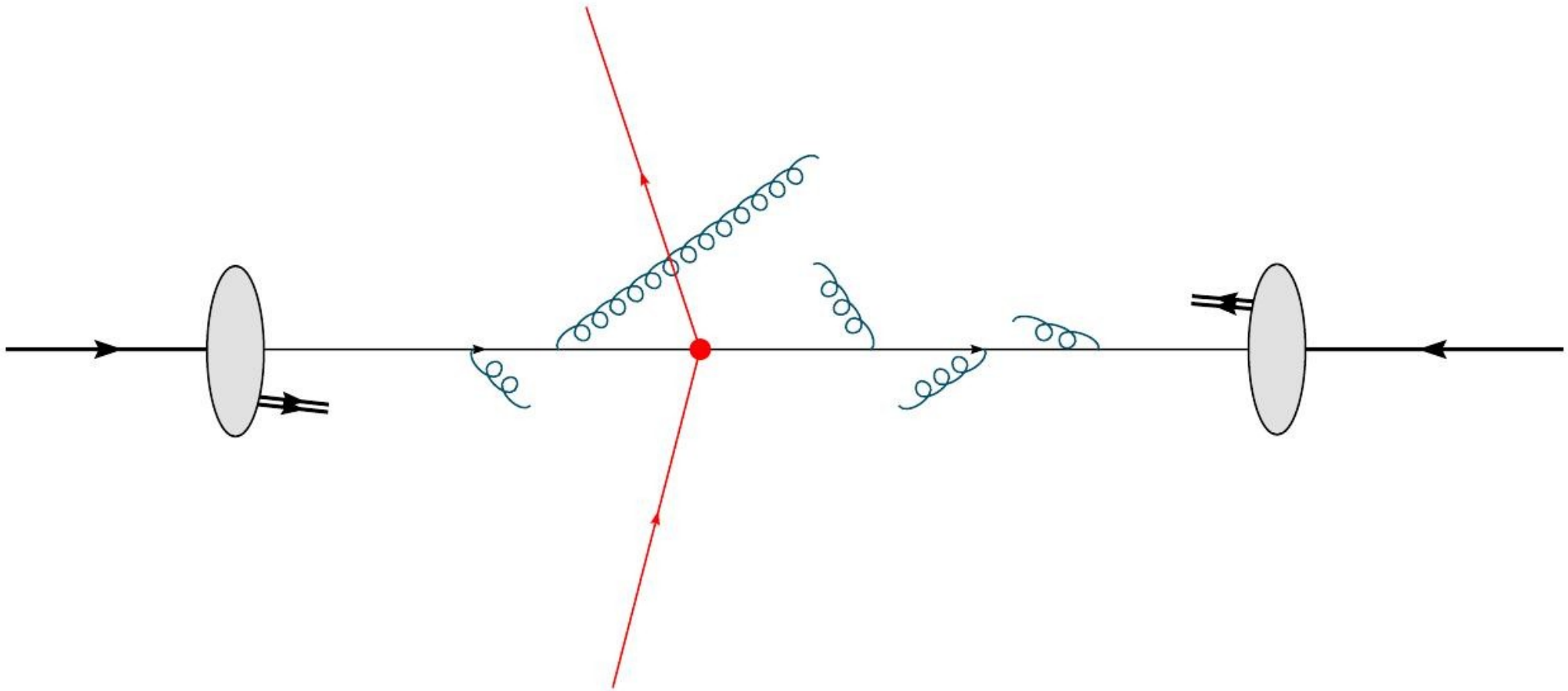
Basic building blocks of Monte Carlo Event Generators

Hard process (exact fixed-order perturbation theory)



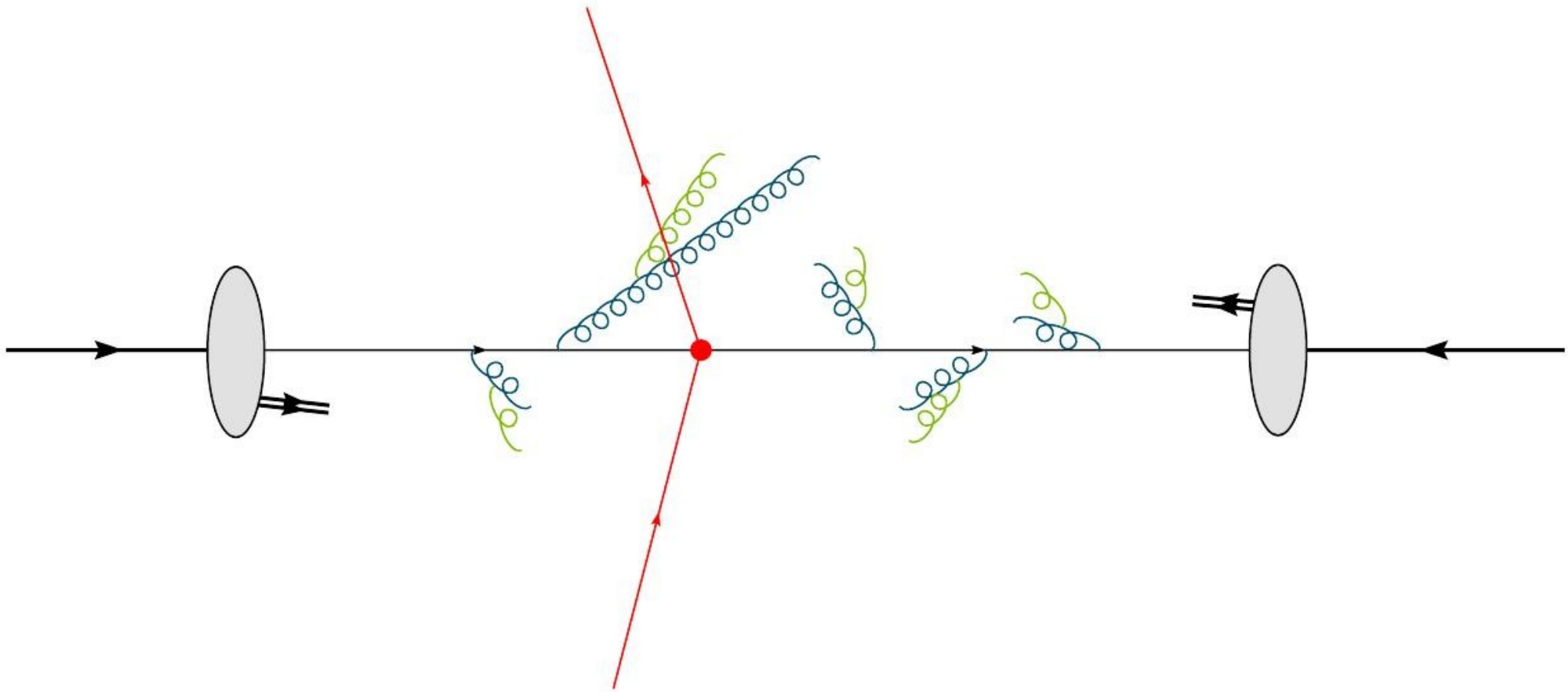
Basic building blocks of Monte Carlo Event Generators

Parton Shower (Approximate all-order perturbation theory)



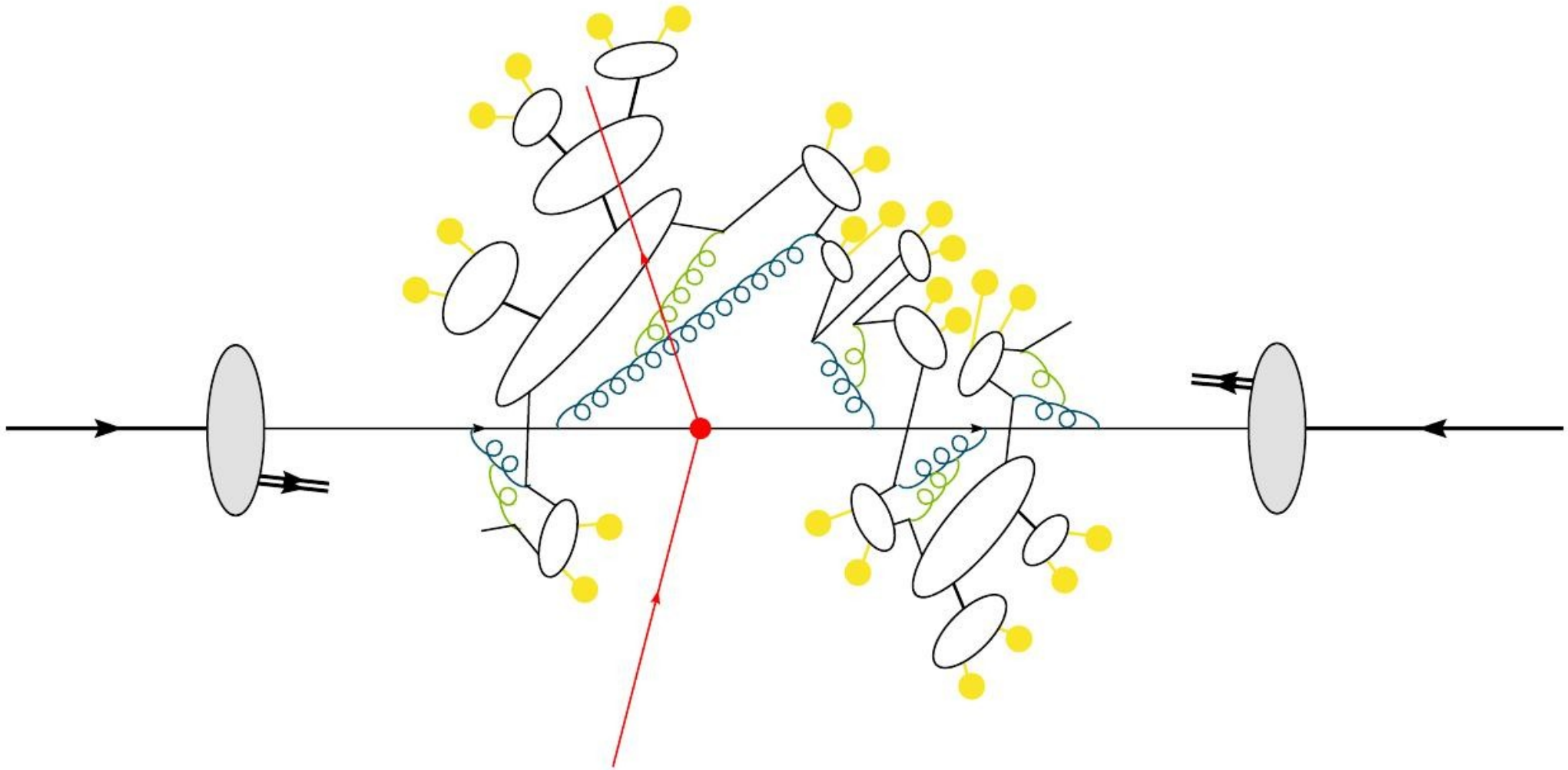
Basic building blocks of Monte Carlo Event Generators

Parton Shower (Approximate all-order perturbation theory)



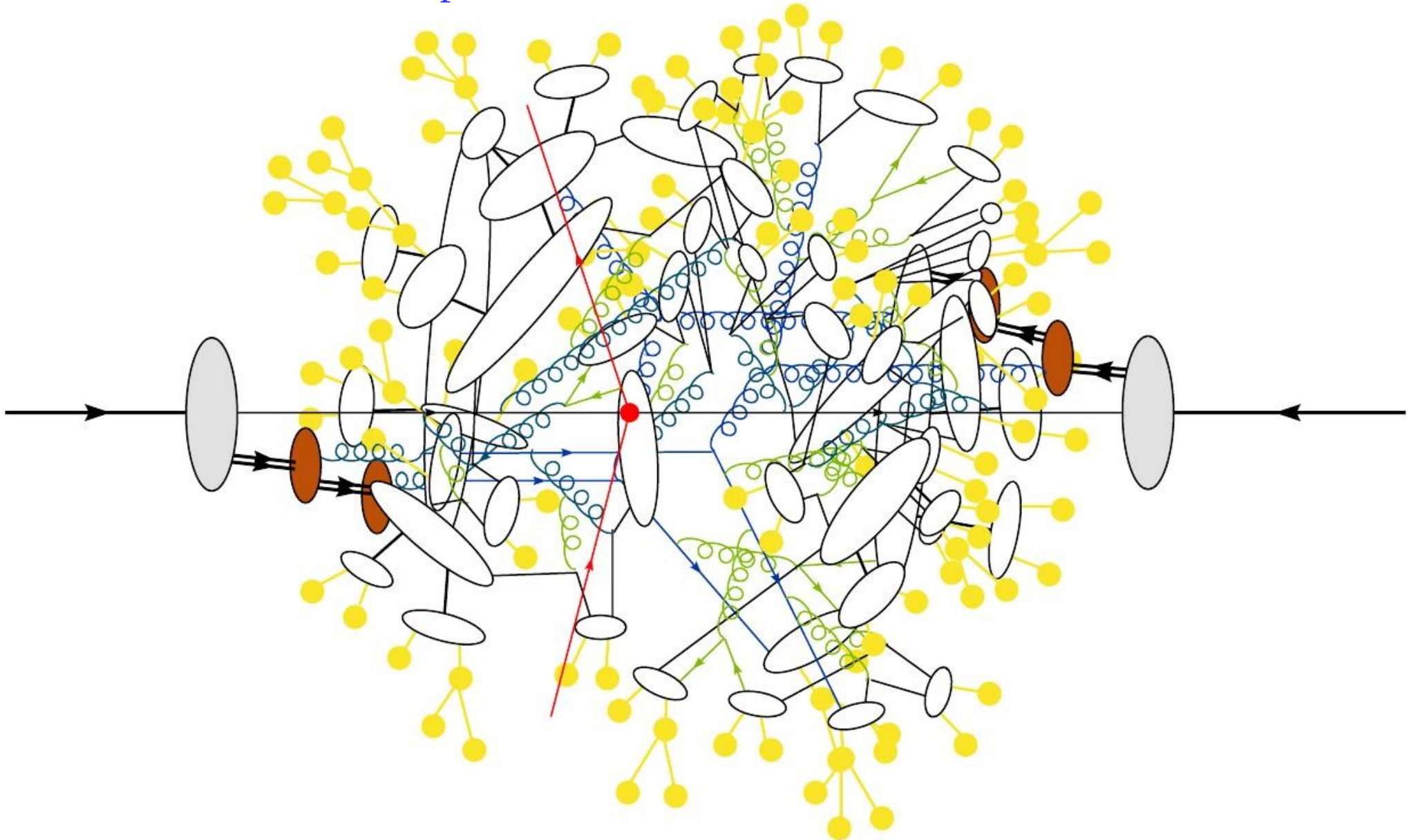
Basic building blocks of Monte Carlo Event Generators

Hadronization (non-perturbative semi-empirical models)



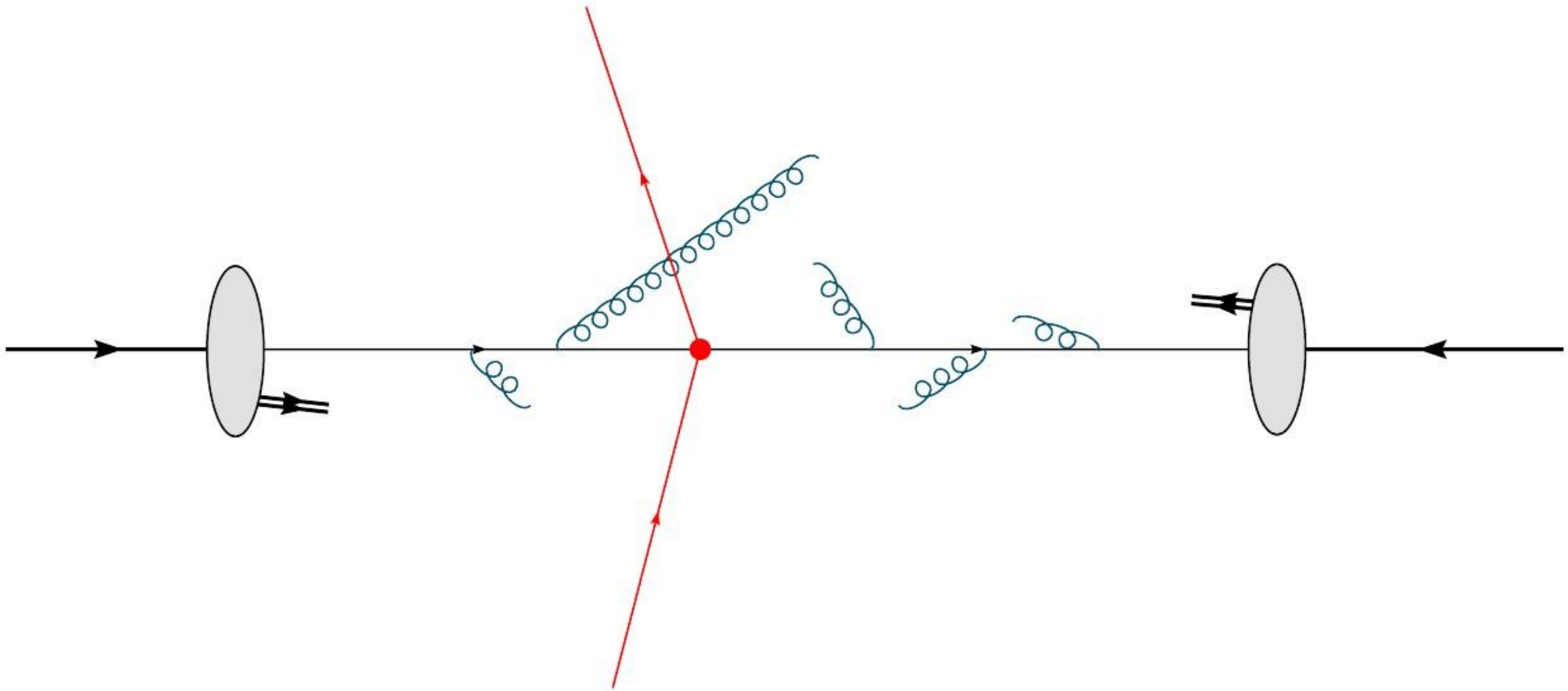
Basic building blocks of Monte Carlo Event Generators

Multiple Interactions and beam remnants



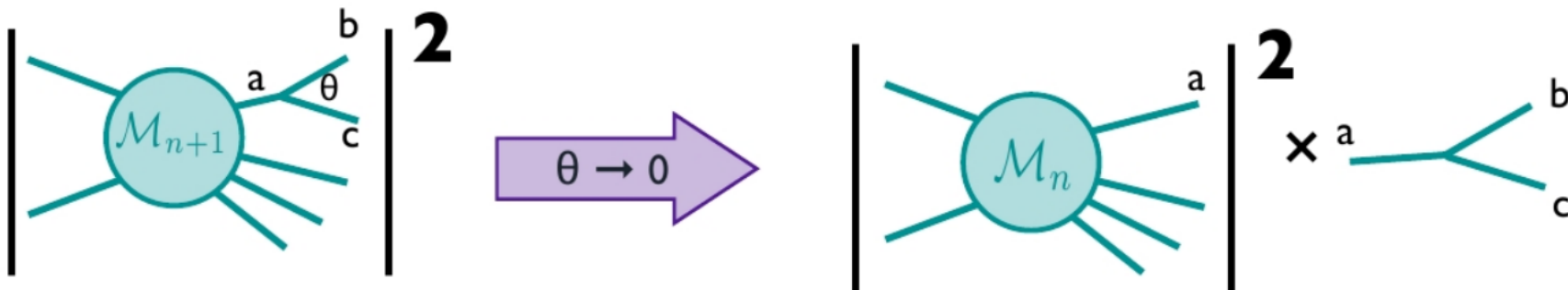
Basic building blocks of Monte Carlo Event Generators

Parton Shower (Approximate all-order perturbation theory)

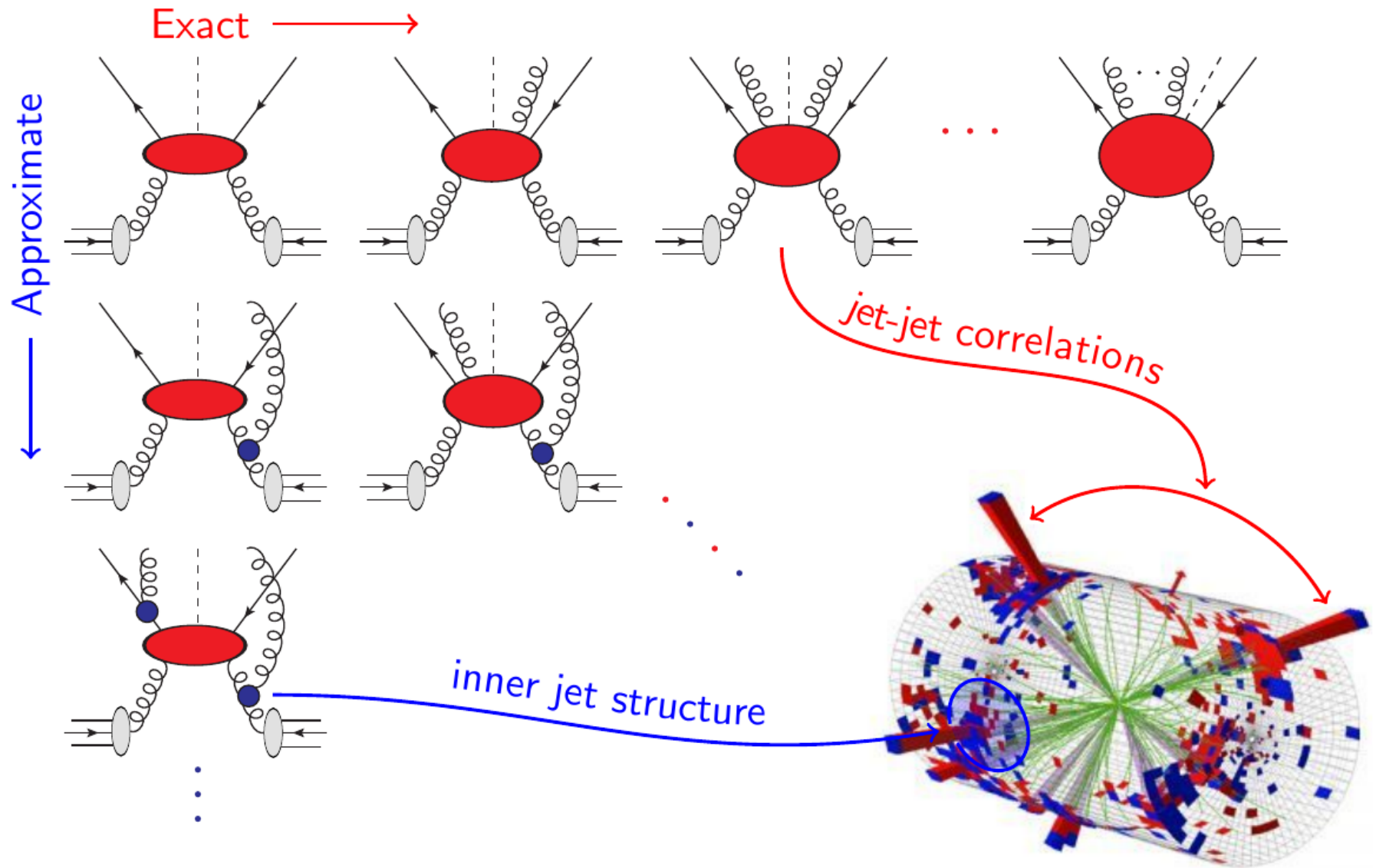


Parton Shower

- ▶ The hard subprocess, by definition, involves large momentum transfers and therefore the partons involved in it are violently accelerated.
- ▶ The accelerated coloured partons will emit QCD radiation in the form of gluons leading to parton showers.
- ▶ In principle, the showers represent higher-order corrections to the hard subprocess. However, it is not feasible to calculate these corrections exactly. Instead, an approximation scheme is used, in which the dominant contributions are included in each order. Large logs $\alpha_s^n \log^{2n} \frac{Q_{hard}}{Q_0 \sim 1 \text{ GeV}} \sim 1$
- ▶ These dominant contributions are associated with collinear parton splitting or soft (low-energy) gluon emission.
- ▶ The conventional parton-shower formalism is based on collinear factorization



Parton Shower



PS is process-independent, however lets start with simple example:

- ▶ Consider $e^+e^- \rightarrow 3$ partons

$$\frac{1}{\sigma_{2 \rightarrow 2}} \frac{d\sigma_{2 \rightarrow 3}}{d\cos\theta dz} \sim C_F \frac{\alpha_s}{2\pi} \frac{2}{\sin^2\theta} \frac{1 + (1-z)^2}{z}$$

θ - angle of gluon emission

z - fractional energy of gluon

- ▶ Divergent in

- ▶ Collinear limit: $\theta \rightarrow 0, \pi$
- ▶ Soft limit: $z \rightarrow 0$

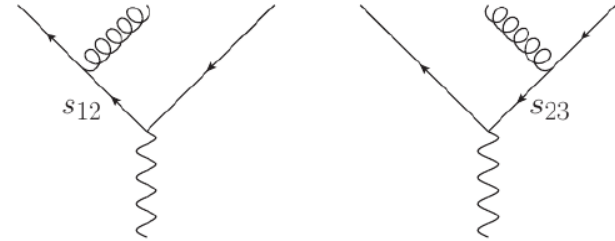
- ▶ Separate into two independent jets

$$\frac{2d\cos\theta}{\sin^2\theta} = \frac{d\cos\theta}{1-\cos\theta} + \frac{d\cos\theta}{1+\cos\theta} = \frac{d\cos\theta}{1-\cos\theta} + \frac{d\cos\bar{\theta}}{1-\cos\bar{\theta}} \approx \frac{d\theta^2}{\theta^2} + \frac{d\bar{\theta}^2}{\bar{\theta}^2}$$

- ▶ Independent jet evolution

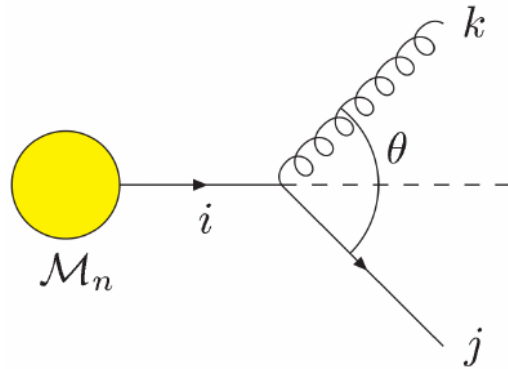
$$d\sigma_3 \sim \sigma_2 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} dz \frac{1 + (1-z)^2}{z}$$

It starts to look like we can iterate it!



- In the collinear limit the cross section for a process factorizes:

$$d\sigma_{n+1} \approx d\sigma_n \frac{\alpha_S}{2\pi} \frac{d\theta^2}{\theta^2} dz d\phi P_{ji}(z, \phi)$$

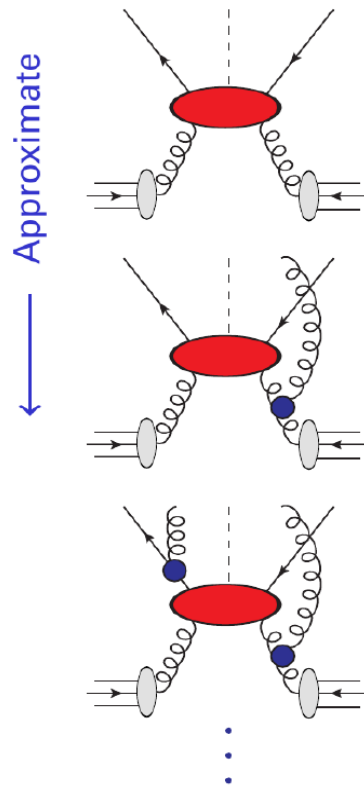


- P_{ji} is the splitting function.
- This is singular as $\theta \rightarrow 0$
- $\frac{d\theta^2}{\theta^2} = \frac{dQ^2}{Q^2} = \frac{dk_{\perp}^2}{k_{\perp}^2} = \frac{dq^2}{q^2}$

- The Sudakov Form Factor Probability of not emitting resolvable radiation

$$\Delta_i(q_1^2, q_2^2) = \exp \left\{ - \int_{q_2^2}^{q_1^2} \frac{dq^2}{q^2} \frac{\alpha_S}{2\pi} \int_{Q_0^2/q^2}^{1-Q_0^2/q^2} dz \int_0^{2\pi} d\phi P_{ji}(z, \phi) \right\} .$$

- The dominant region of phase space is the one where radiation is strongly ordered in evolution variable q .
- Many choices of q are equivalent for collinear-enhanced contributions but they differ in soft gluon emission, which is also enhanced.
- Within the conventional parton-shower formalism, based on collinear factorization, it was shown that the soft region can be correctly described by using the angle of the emissions (Herwig) as the ordering

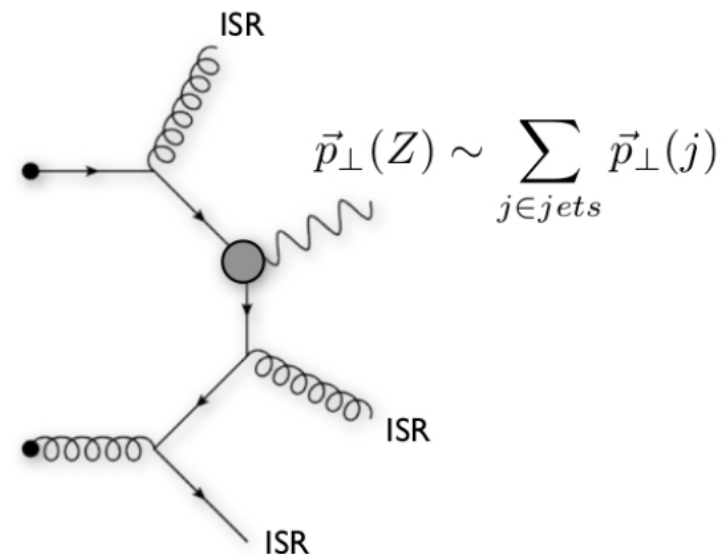
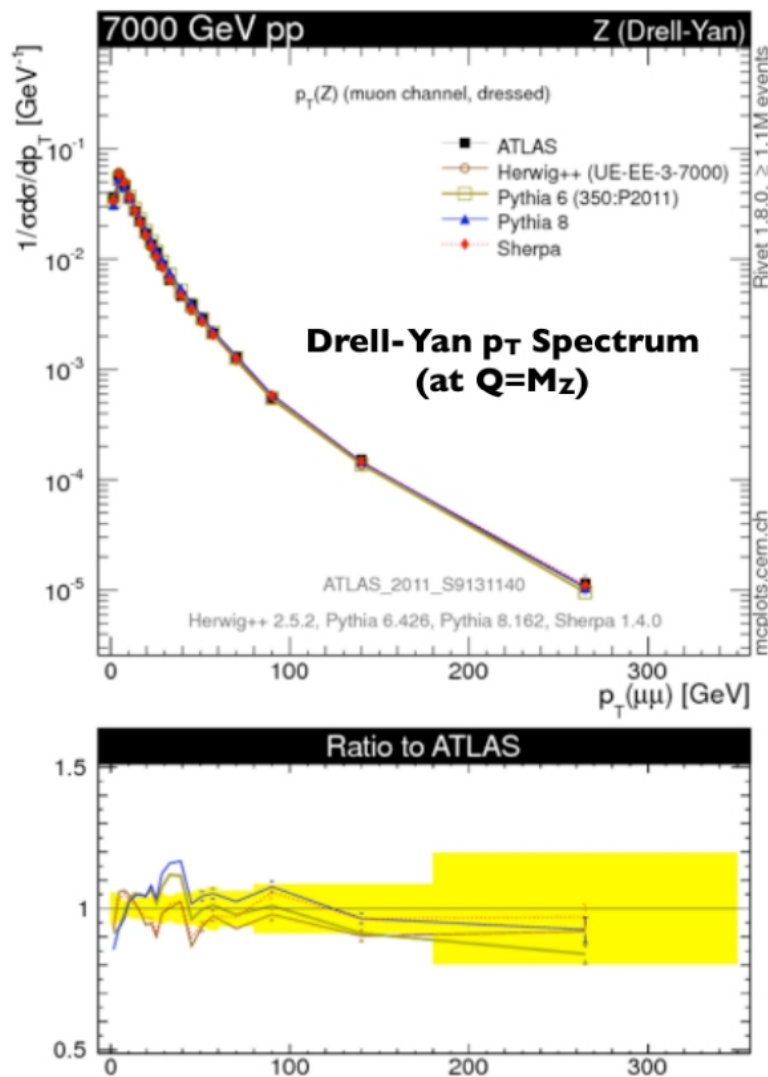


$$\sigma_{\text{incl}} \left[\Delta(t_0, \mu_Q^2) \right.$$

$$+ \int_{t_0}^{\mu_Q^2} \frac{dt}{t} \int dz \frac{\alpha_s}{2\pi} P(z) \Delta(t, \mu_Q^2)$$

$$+ \frac{1}{2} \left(\int_{t_0}^{\mu_Q^2} \frac{dt}{t} \int dz \frac{\alpha_s}{2\pi} P(z) \right)^2 \Delta(t, \mu_Q^2)$$

$$+ \dots$$

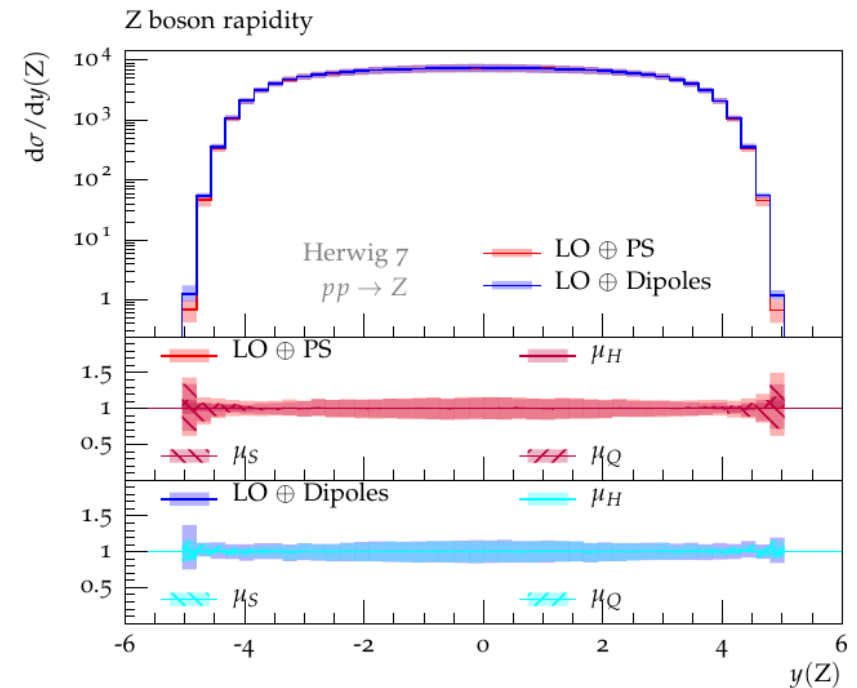
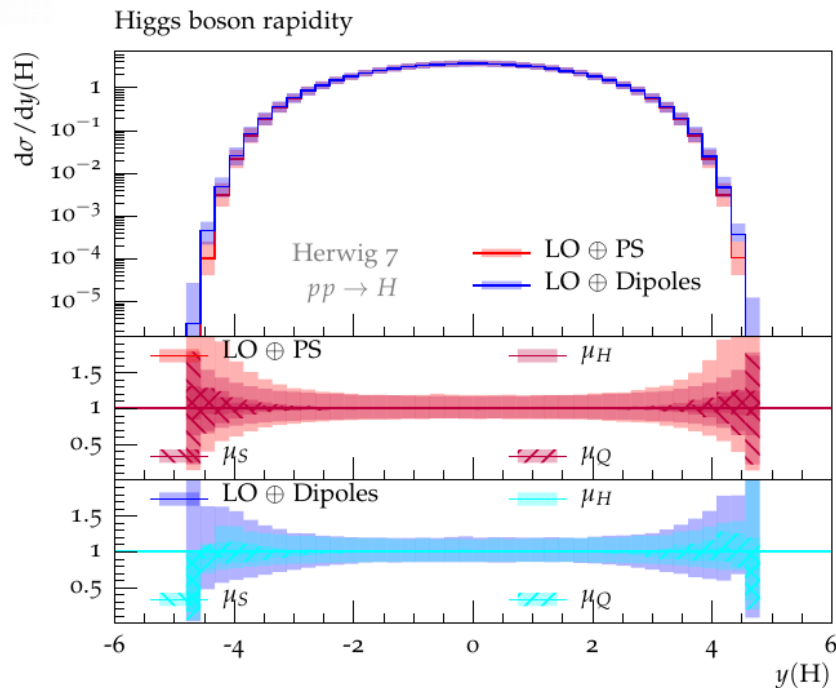


Particularly sensitive to

1. α_s renormalization scale choice
2. Recoil strategy (color dipoles vs global vs ...)
3. FSR off ISR (ISR jet broadening)

Non-trivial result that modern GPMC shower models all reproduce it \sim correctly

Note: old PYTHIA 6 model (Tune A) did not give correct distribution, except with extreme μ_R choice (DW, D6, Pro-Q20)



Two Parton Showers:

- ▶ Angular-ordered Parton Shower (PS)
- ▶ p_T -ordered Dipole Shower

Up/Down Variations of:

- ▶ μ_H - argument of PDF, α_S in hard matrix element
- ▶ μ_S - argument of PDF, α_S in the shower
- ▶ μ_Q - shower starting/veto scale
- ▶ μ_{IR} - shower cutoff

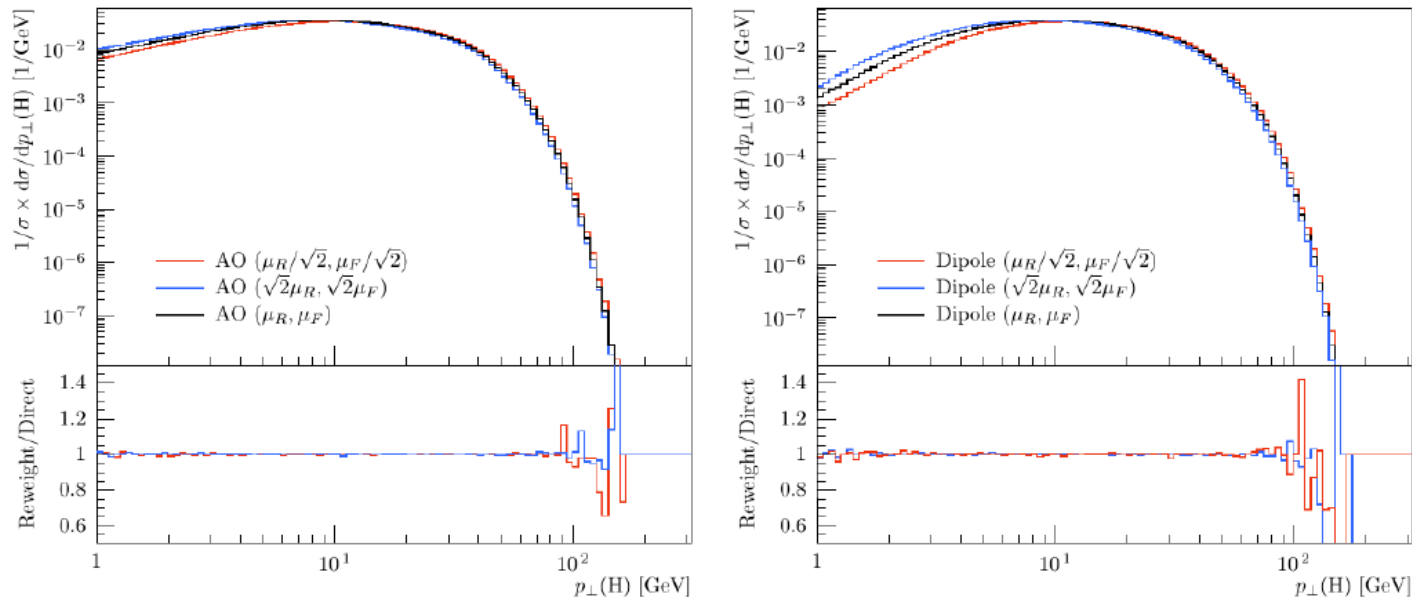
Parton Shower uncertainties

Run-time improvement via parton-shower reweighting

[Bellm, Plätzer, Richardson, Siodmok, Webster, Phys.Rev. D94 (2016)]

Transverse momentum of Higgs boson in $pp \rightarrow gg \rightarrow H$, $\sqrt{S} = 13$ TeV

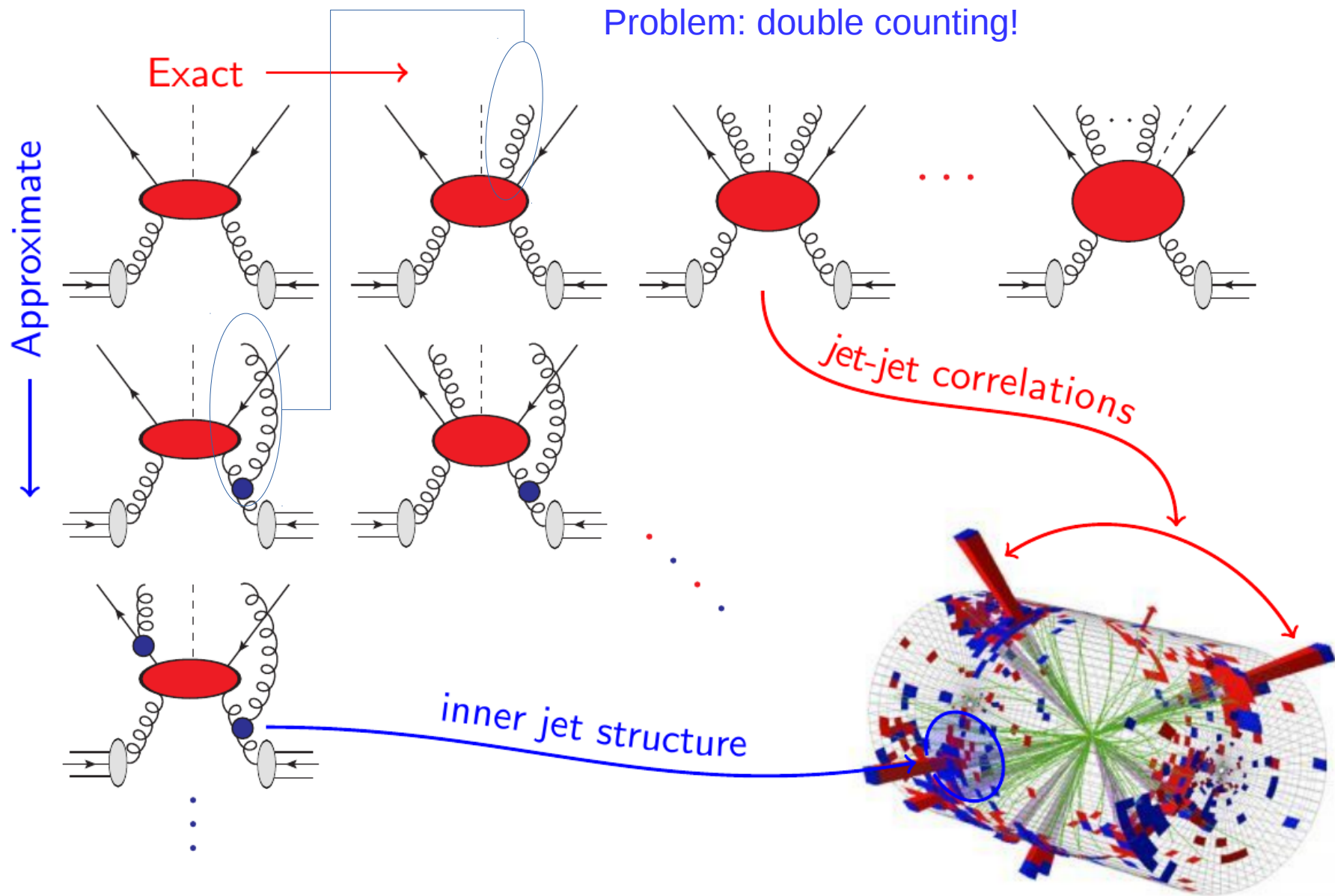
H7



- ▶ excellent agreement between individual runs for different scales and reweighting
- ▶ significant speed improvements: time in seconds for 10 000 events

Shower	Hadron-ization & Decays	No MPI			MPI					
		Direct	Reweight	Frac. Diff.	Direct	Primary Reweight	Frac. Diff.	Direct	All Reweight	Frac. Diff.
AO	Off	79.8	94.2	-0.18	384.4	249.1	0.35	416.7	375.1	0.09
	On	183.2	128.3	0.30	738.7	364.3	0.51	751.4	482.3	0.35
Dipole	Off	99.6	52.8	0.47	435.4	161.9	0.63	462.7	213.6	0.54
	On	271.8	108.2	0.60	831.7	286.6	0.65	859.2	340.1	0.60

Parton Shower + NLO Matrix Element Matching



Parton Shower + NLO Matrix Element Matching

- NLO matched to parton showers as new default. Matching mechanism fully generic, fully automated for two showers and two matching schemes [subtractive (MC@NLO-type) multiplicative (Powheg-type)]

H7

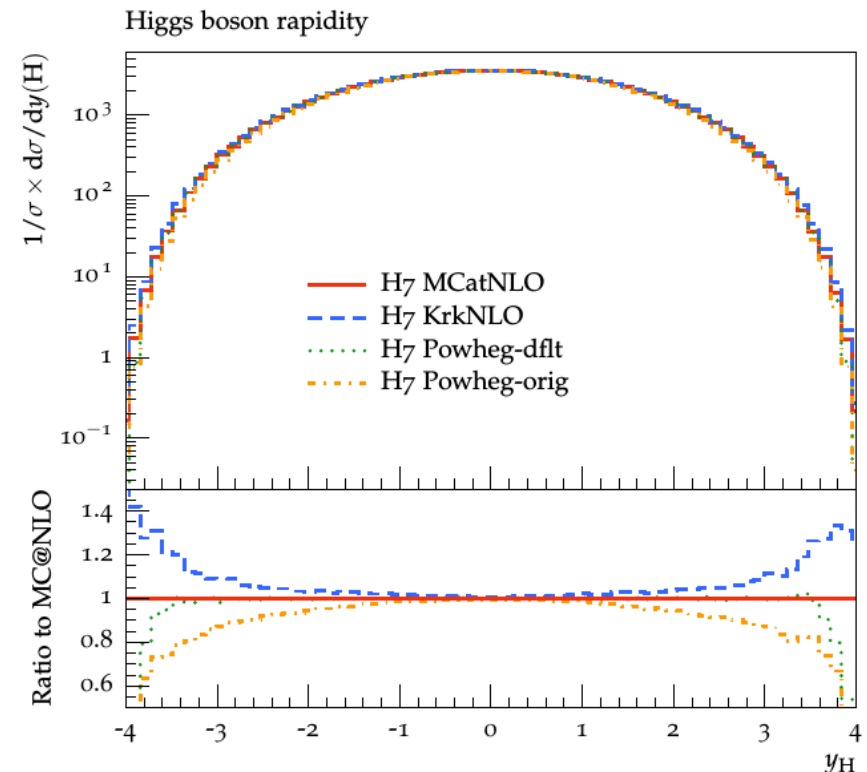
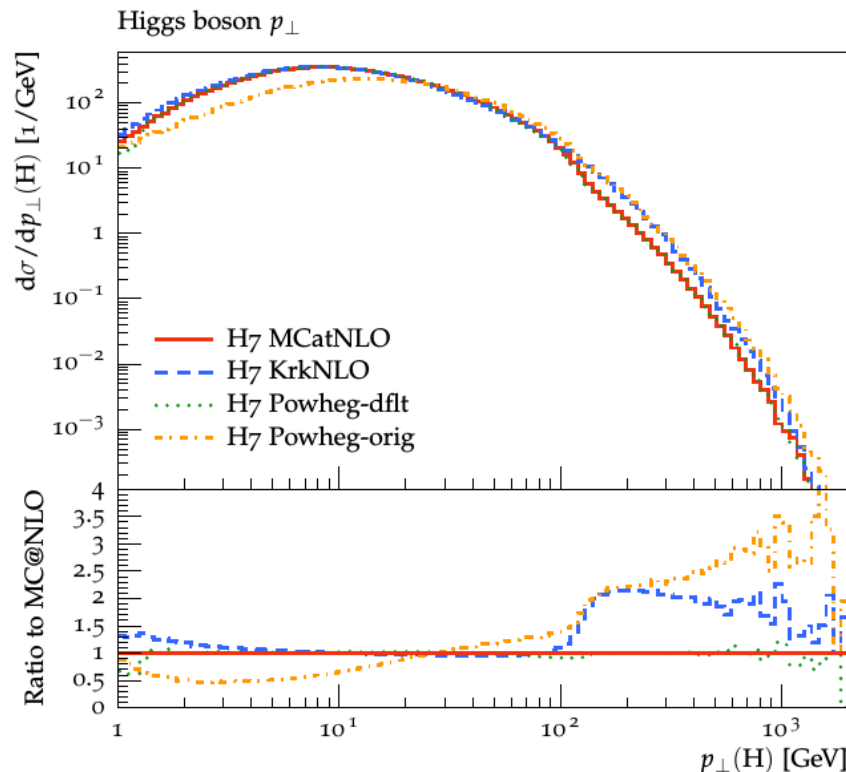
[Bellm, Gieseke, Grellscheid, Plätzer, Rauch, Reuschle, Richardson, Schichtel, Seymour, Siodmok, Wilcock, Fischer, Harrendorf, Nail, Papaefstathiou, D. Rauch, Eur.Phys.J. C76 (2016)]

- Also a new matching method **KrkNLO** available in H7

[Jadach, Pałczek, Sapeta, Siódmok, Skrzypek. JHEP 1510 (2015)]

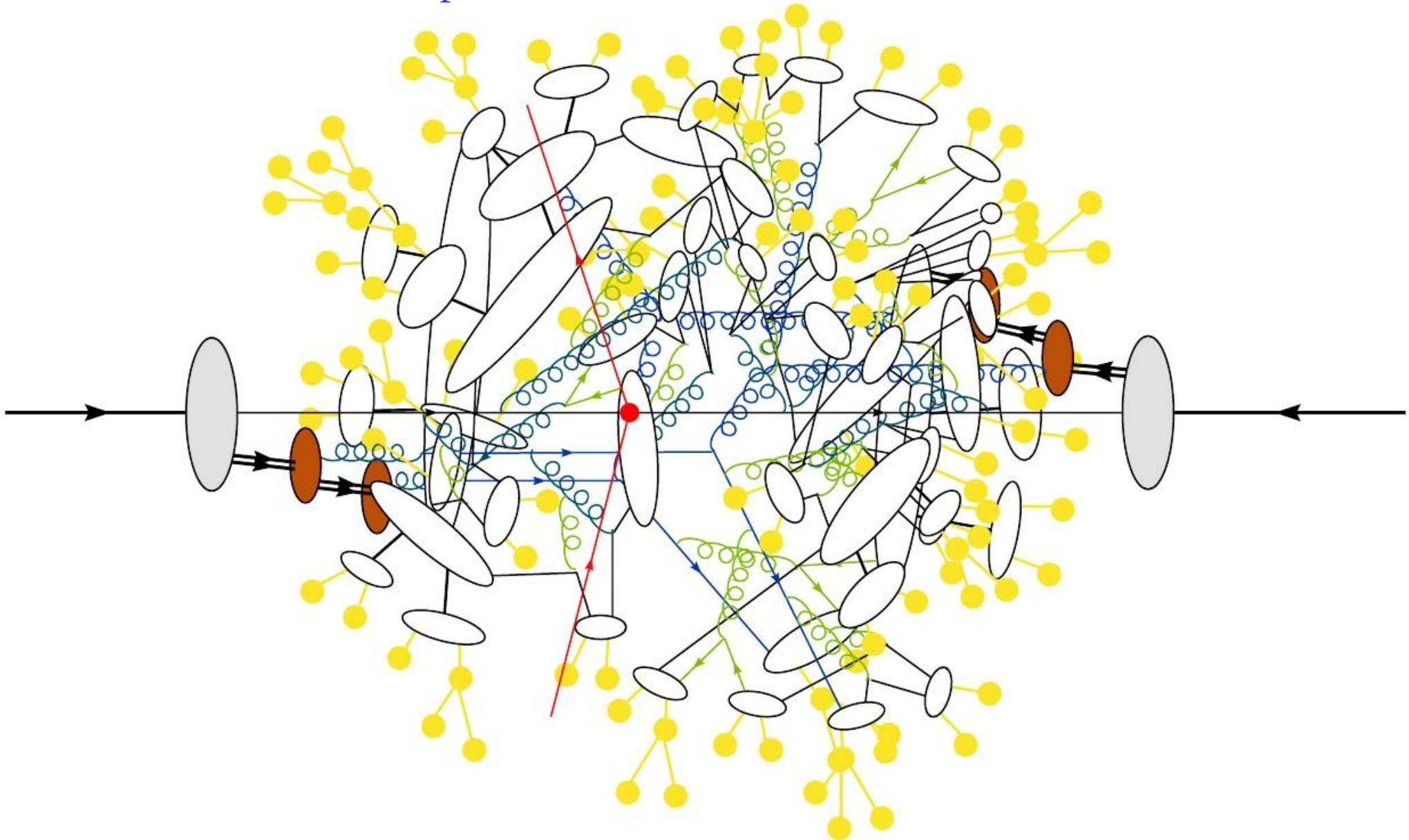
[Jadach, Pałczek, Sapeta, Siódmok, Skrzypek. 76, no. 12, 649 (2016)]

[Jadach, Nail, Pałczek, Sapeta, Siodmok, Skrzypek. Eur. Phys. J. C 77 (2017)]



Basic building blocks of Monte Carlo Event Generators

Multiple Interactions and beam remnants



How do we know MPI exists? Data makes you smarter!

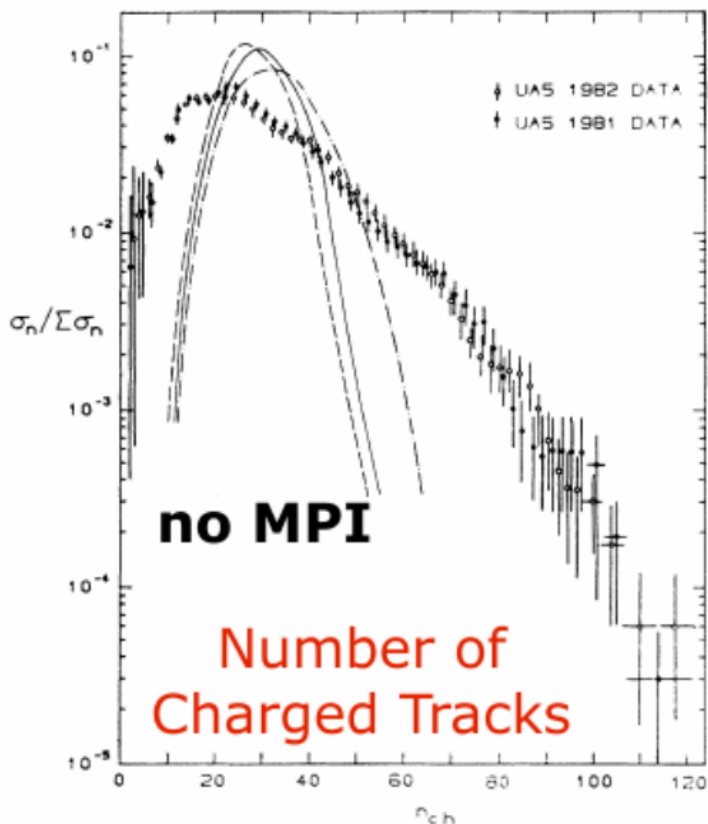


FIG. 3. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs simple models: dashed low p_T only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.

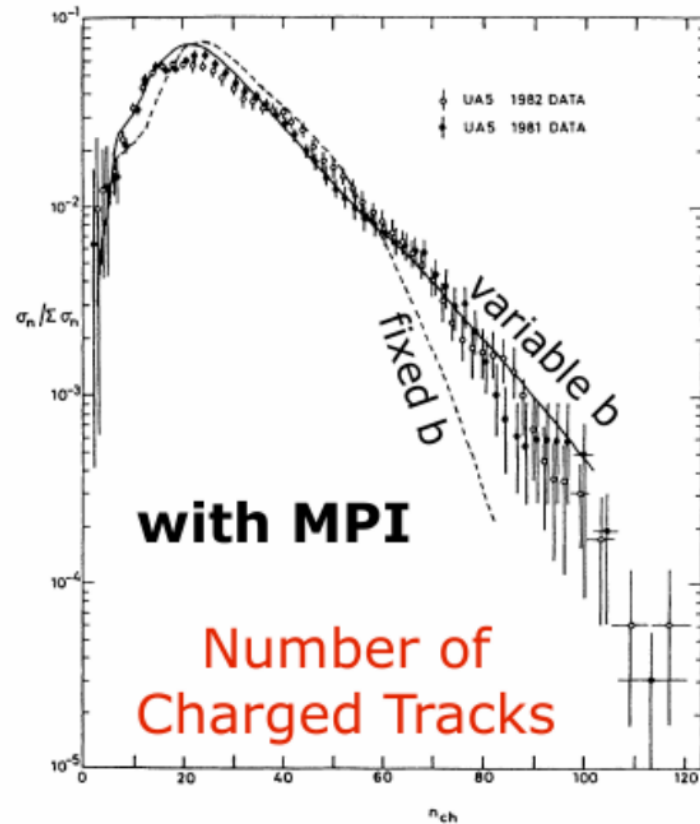


FIG. 12. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs multiple-interaction model with variable impact parameter: solid line, double-Gaussian matter distribution; dashed line, with fix impact parameter [i.e., $\bar{O}_0(b)$].

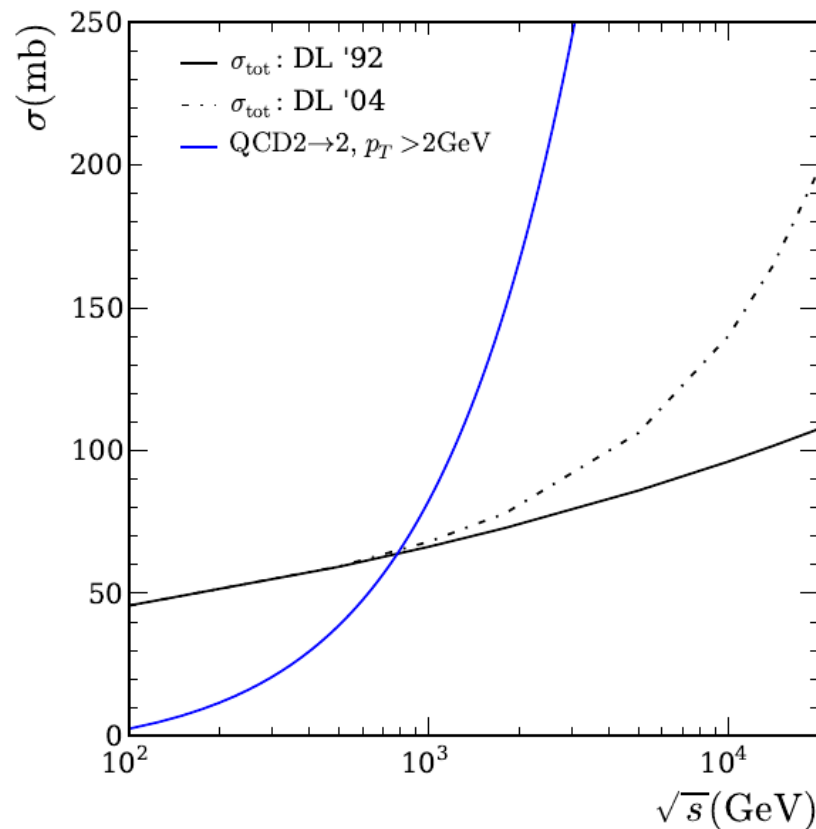
Sjöstrand & v. Zijl,
Phys.Rev.D36(1987)2019

Motivation:

- ▶ The minimum bias / underlying event is an unavoidable background to most collider observables and having good understand of it leads to more precise collider measurements!
- ▶ First LHC results are Minimum Bias and Underlying Event! Alice: [0911.5430], CMS [1002.0621], ATLAS [1003.3124] so it must be important ;)
- ▶ These will be particularly relevant for the LHC as, when it is operated at design luminosity, rare signal events will be embedded in a background of more than 20 near-simultaneous minimum-bias collisions.
- ▶ Any realistic experiment simulation event generator needs to be able to model these effects.
- ▶ “Don’t worry, we will measure and subtract it” But... fluctuations and correlations on an event-by-event basis are crucial.

Inclusive hard jet cross section in pQCD:

$$\sigma^{\text{inc}}(s, p_t^{\text{min}}) = \sum_{i,j} \int_{p_t^{\text{min}2}} dp_t^2 \int dx_1 dx_2 f_i(x_1, Q^2) f_j(x_2, Q^2) \frac{d\hat{\sigma}_{ij}}{dp_t^2}$$



$\sigma^{\text{inc}} > \sigma_{\text{tot}}$ eventually

Interpretation:

- ▶ σ^{inc} counts **all** partonic scatters in a single pp collision
- ▶ more than a single interaction

$$\sigma^{\text{inc}} = \langle n_{\text{dijets}} \rangle \sigma_{\text{inel}}$$

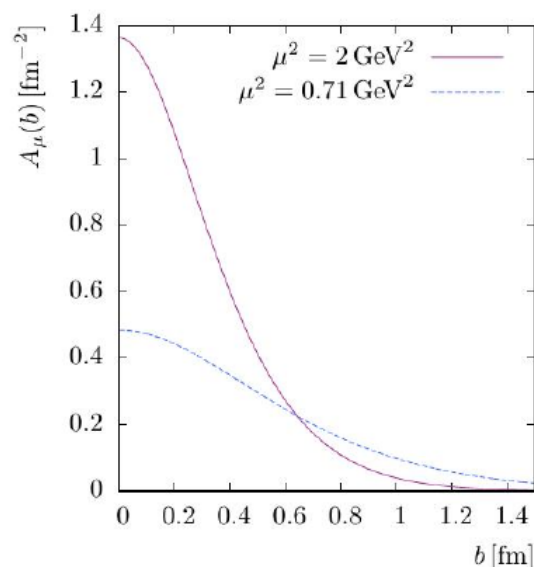
Assumptions:

- the distribution of partons in hadrons factorizes with respect to the b and x dependence \Rightarrow average number of parton collisions:

$$\begin{aligned}
 \bar{n}(\vec{b}, s) &= L_{\text{partons}}(x_1, x_2, \vec{b}) \otimes \sum_{ij} \int dp_t^2 \frac{d\hat{\sigma}_{ij}}{dp_t^2} \\
 &= \sum_{ij} \frac{1}{1 + \delta_{ij}} \int dx_1 dx_2 \int d^2\vec{b}' \int dp_t^2 \frac{d\hat{\sigma}_{ij}}{dp_t^2} \\
 &\quad \times D_{i/A}(x_1, p_t^2, |\vec{b}'|) D_{j/B}(x_2, p_t^2, |\vec{b} - \vec{b}'|) \\
 &= \sum_{ij} \frac{1}{1 + \delta_{ij}} \int dx_1 dx_2 \int d^2\vec{b}' \int dp_t^2 \frac{d\hat{\sigma}_{ij}}{dp_t^2} \\
 &\quad \times f_{i/A}(x_1, p_t^2) G_A(|\vec{b}'|) f_{j/B}(x_2, p_t^2) G_B(|\vec{b} - \vec{b}'|) \\
 &= A(\vec{b}) \sigma^{\text{inc}}(s; p_t^{\text{min}}) .
 \end{aligned}$$

- at fixed impact parameter b , individual scatterings are independent (leads to the Poisson distribution)

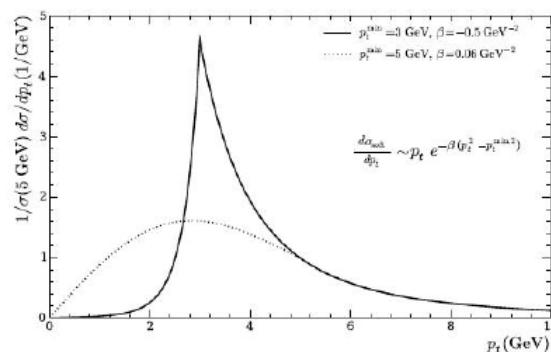
Matter distribution (μ^2)



Based on electromagnetic form factor
(radius of the proton free parameter)

Extension to soft MPI

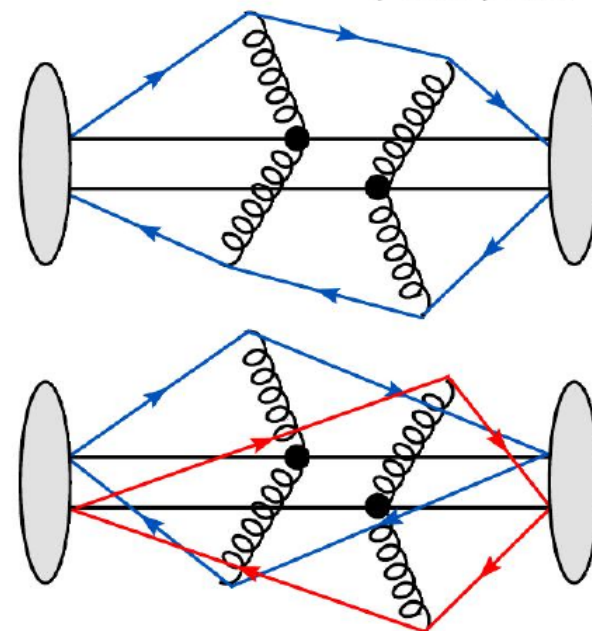
($p_t < p_t^{\min}$)



Gaussian extension below p_t^{\min}

Energy dependent p_t^{\min}

Colour structure (p_{reco}, p_{CD})



Possibility of change of color structure
(color reconnection)

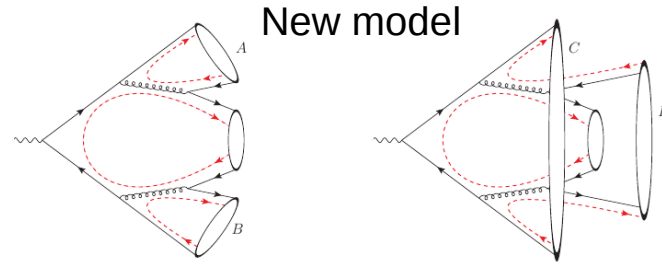
The least understood part of modeling

Main parameters:

- ▶ μ^2 - inverse hadron radius squared (parametrization of overlap function)
- ▶ p_t^{\min} - transition scale between soft and hard components $\Rightarrow p_t^{\min} = p_{t,0}^{\min} \left(\frac{\sqrt{s}}{E_0} \right)^b$
- ▶ p_{reco} - colour reconnection

[S. Gieseke, Roher, Siodmok EPJC C72 (2012)]

MPI – constraining the models

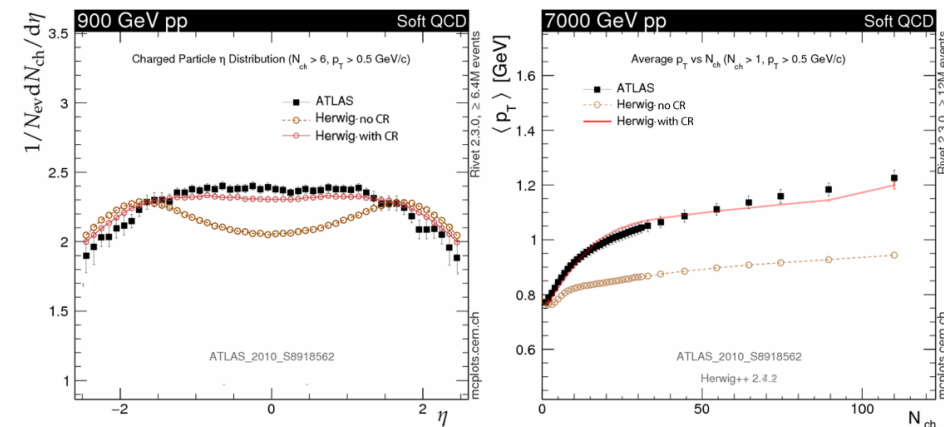


[S. Gieseke, Roher, Siodmok EPJC C72 (2012)]

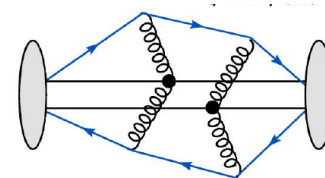
Fix parameters using data

Constrain models even more

[Seymour, Siodmok. JHEP 1310, 113 (2013)]



Correct CDF measurement

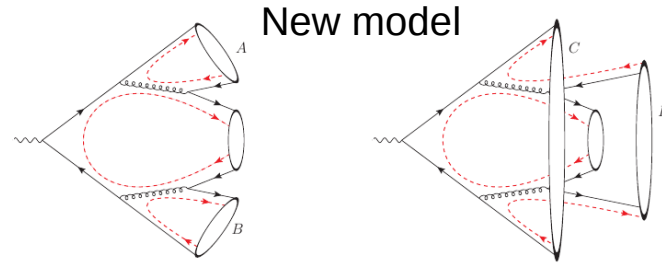


CDF data on double parton interaction

Find more data

[Bahr, Myska, Seymour, Siodmok JHEP 1303, 129 (2013)]

MPI – constraining the models

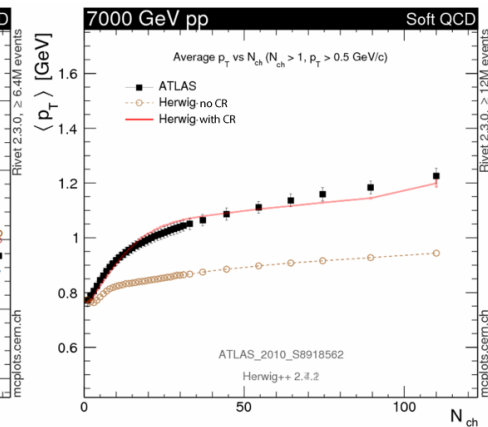
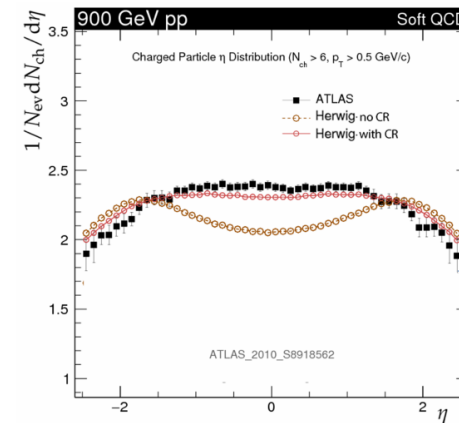


[Gieseke, Roher, Siodmok EPJC C72 (2012)]
[Gieseke, Kirchga  er, Pl  tzer, Siodmok,
submitted to JHEP] ...

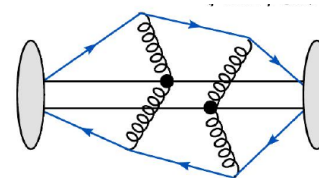
Fix parameters using
data

Constrain models even more

[Seymour, Siodmok. JHEP
1310, 113 (2013)]



Correct CDF
measurement



CDF data on double
parton interaction

Find more data

[Bahr, Myska, Seymour, Siodmok
JHEP 1303, 129 (2013)]

How we improve the models

Idea use q/g jets for BSM search



Construct new observables
sensitive q/g jets
[Baron, Siodmok]

Improve description of q/g
jets in MCEG

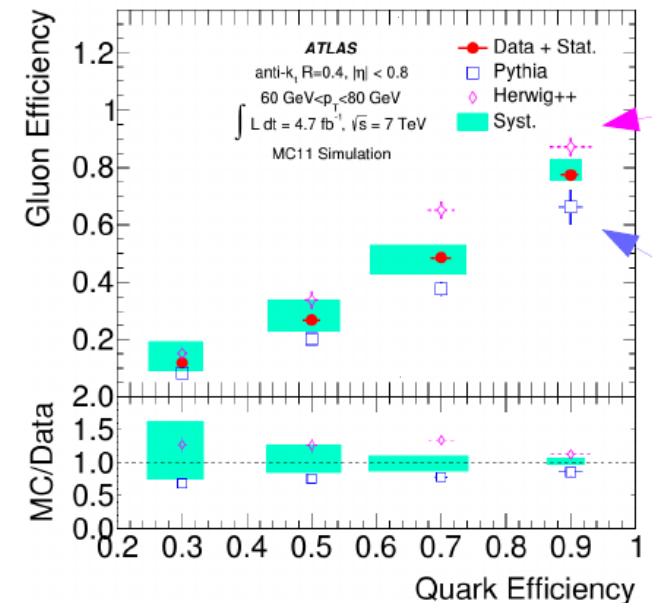
[Reichelt, Richardson,
Siodmok EPJC C77 (2017)]



Study this problem in more details

[Gras, Höche, Kar, Larkoski, Lönnblad, Plätzer, Siódmok,
Skands, Soyez, Thaler JHEP, 1707, 091 (2017)]

Validate the idea against the data



- “Virtual Colliders” = Monte Carlo Event Generators (GPMC)
- Almost all HEP measurements and discoveries in the modern era have relied on GPMC generators, most notably the discovery of the Higgs boson.
- Complex structure of Quantum Chromodynamics we need:
 - Perturbative techniques (hard process – “NLO revolution”)
 - Resummation techniques (Parton Shower – well established)
 - Non-perturbative models (crucial to obtain fully exclusive simulation of the collisions)
- GPMC encodes the global picture of previous measurements
- Tremendous amount of new developments in GPMCs because we need more precise results.
- Good first round of LHC data well described...
- ... but still a lot of space for improvements.

- ▶ Event generators crucial since the start of LHC studies.
- ▶ Qualitatively predictive already 25 years ago
- ▶ Quantitatively steady progress, continuing today:
 - ▶ continuous dialogue with experimental community,
 - ▶ more powerful computational techniques and computers,
 - ▶ new ideas.
- ▶ As LHC needs to study more rare phenomena and more subtle effects, generators must keep up by increased precision.
- ▶ If you are interested to learn more just drop by to my office 4205 :)

- A. Siodmok: “CTEQ School” Pittsburgh 2017
“DESY MC School”, Hamburg 2018



Literature

- R. K. Ellis, W. J. Stirling, B. R. Webber
QCD and Collider Physics
Cambridge University Press, 2003
- T. Sjostrand, S. Mrenna, P. Z. Skands
Pythia 6.4 Physics and Manual
JHEP 05 (2006) 026
- A. Buckley et al.
General-purpose event generators for LHC physics
Phys. Rept. 504 (2011) 145
- A. Siodmok
LHC Event Generation with General-purpose Monte Carlo Tools
Acta Phys.Polon. B44 (2013) no.7, 1587-1601

Monte Carlo

training studentships



3-6 month fully funded studentships for current PhD students at one of the MCnet nodes. An excellent opportunity to really understand and improve the Monte Carlos you use!

Application rounds every 3 months.

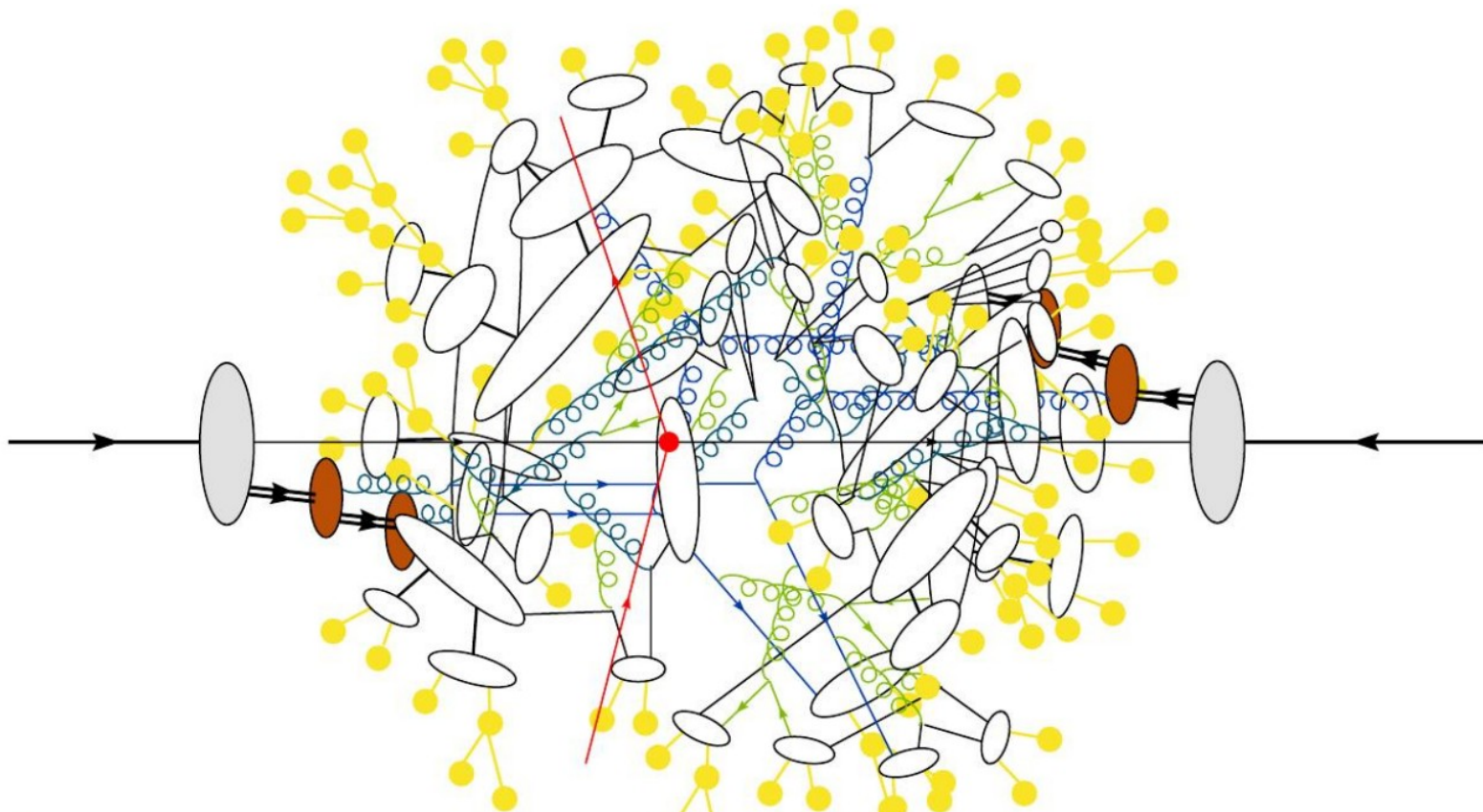
MCnet projects
Pythia+Vincia
Herwig
Sherpa
MadGraph
“Plugin” – Ariadne+HEJ
CEDAR – Rivet+Professor
+Contur+hepforge+...



for details go to:
www.montecarlonet.org

Thank you for your attention!

Monte Carlo methods why and how?



- ▶ We want to compute expectation values of observables
$$\langle O \rangle = \sum_n \int d\phi_n P(\phi_n) O(\phi_n),$$
where ϕ_n - Point in n -particle phase-space, $P(\phi_n)$ Probability to produce ϕ_n , Value of observable at $O(\phi_n)$.
- ▶ large n $\mathcal{O}(100 \div 1000) \Rightarrow$ Monte Carlo is the only choice.

$$\langle O \rangle = \sum_n \int d\phi_n P(\phi_n) O(\phi_n)$$

Problems:

- Integrate a multi dimensional function

Efficiencies of integration methods (MC with numerical quadrature):

Uncertainty as a function of number of points n	In 1 dim.	In d dim.
Monte Carlo	$n^{-1/2}$	$n^{-1/2}$
Trapezoidal rule	n^{-2}	$n^{-2/d}$
Simpson's rule	n^{-4}	$n^{-4/d}$

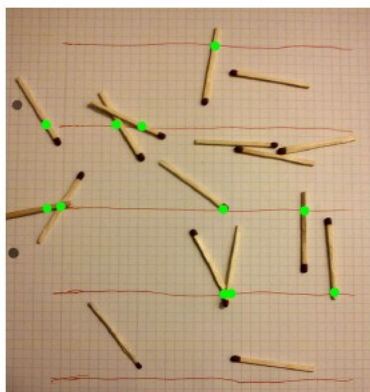
- Pick a point at random according to a probability distribution.

Wikipedia

Monte Carlo methods are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results.

History:

- ▶ G. Comte de Buffon (1777) – perhaps the earliest documented use of random sampling to find the solution to the integral (by throwing a needle onto horizontal plane ruled with straight lines).
- ▶ Marquis Pierre-Simon de Laplace (1886) – use of Buffon's method to evaluate π .



Calculate π by dropping a needle onto the floor.

$\Leftarrow 34/11 \sim 3.1$ based on 17 throws

- ▶ Lord Kelvin (1901) – use random sampling (drawing numbered pieces of paper from a bowl) to aid in evaluating some integrals in the kinetic theory of gases.

History – cont.

- ▶ Enrico Fermi (1930s) – numerical sampling experiments on neutron diffusion and transport in nuclear reactors (devised FERMIAC – a mechanical sampling device).



← S. Ulam with FERMIAC

- ▶ J. von Neumann, S. Ulam, N. Metropolis, R. Feynman (1940s) – first large-scale random-numbers based calculations of neutron scattering and absorption during the “Manhattan” project (work on a nuclear bomb). Name Monte Carlo refers to the Monte Carlo Casino in Monaco where Ulam’s uncle would borrow money from relatives to gamble.
- ▶ ...
- ▶ In Particle Physics we have to solve multidimensional integrals (many particles) MC methods very efficient! So we play a roulette to understand the law of the nature :)